Differences between neighborhoods on 2D real time cellular automata

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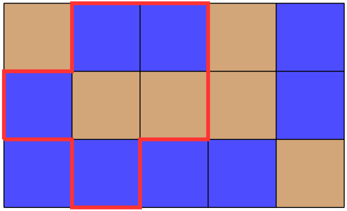
Automata

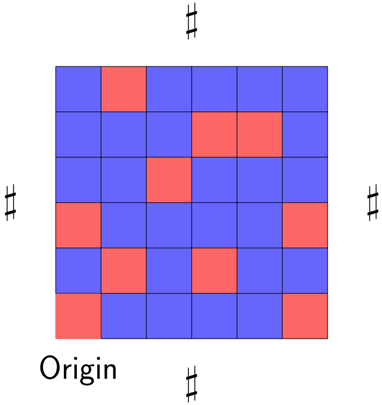
Definition

A *cellular automaton* is a triplet (Q, V, δ) where

- Q is a finite set of states
- V is a finite subset of $\mathbb{Z}^2$ called neighborhood. We consider that $(0,0)$ is always in V.
- δ is a function from Q^V to Q, called the local transition function.

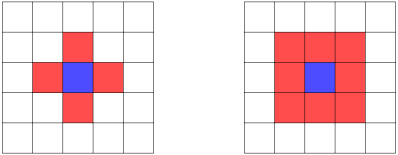


Recognition



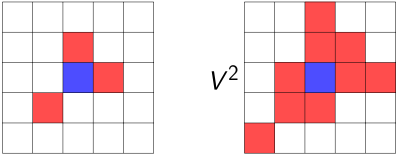
- Parellel input
- Recognition at the origin cell
- Time complexity T(n,m)

Neighborhoods



Von Neumann Moore

V



V^2

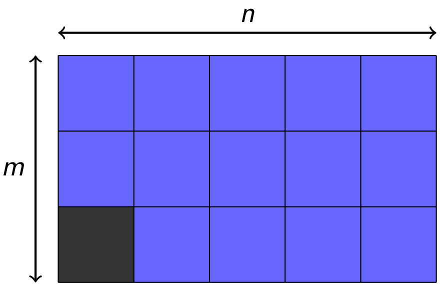
Proposition

V is complete iff $V^\infty = \mathbb{Z}^2$.

Real Time

Definition

The *real time* (TR) is the minimal time such that the state of the outpout cell depends on all the input. It depends both on the neighborhood and on the input size.



$$TR_{VN} = n + m - 1$$

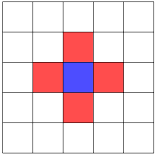
$$TR_M = max(n, m) - 1$$

Linear equivalence

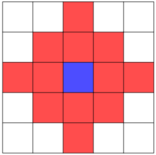
Lemma

For all complete neighborhoods V and U :
If there is an automaton A with neighborhood V recognizing L in time $t(input)$, there exists an automaton B with neighborhood U recognizing L in time $k \times t(input)$.

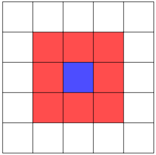
V_{VN}



V_{VN}^2

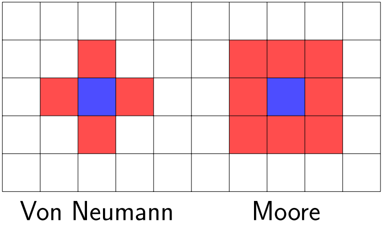


V_M



$$V_M \subset V_{VN}^2$$

Existing Result

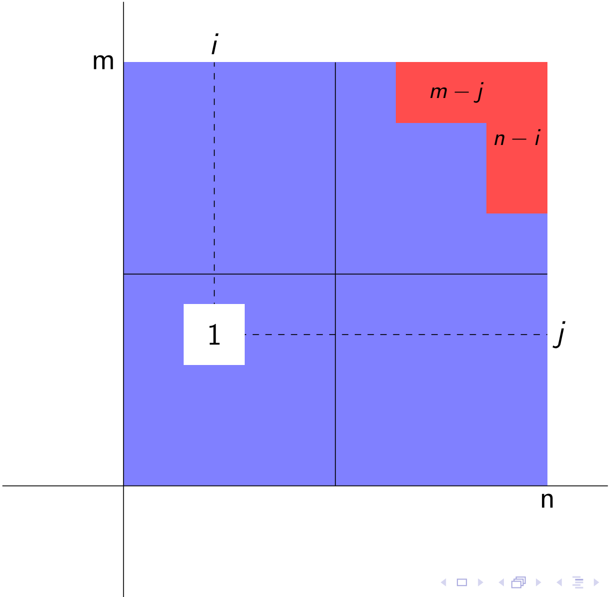


Theorem (Terrier, 1999)

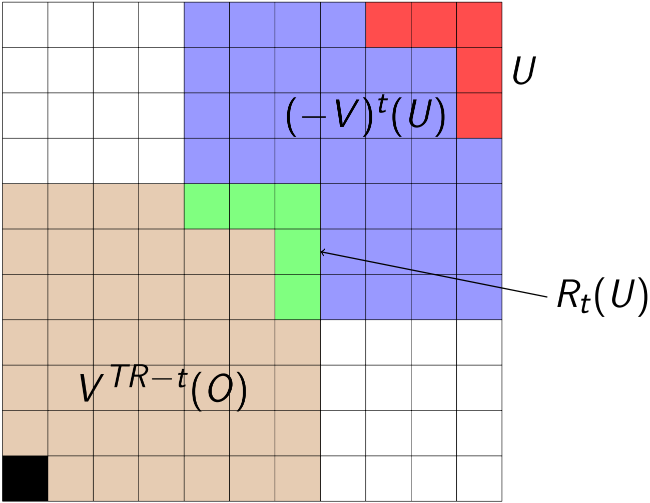
There exists a language L such that :

- *No automaton with Moore Neighborhood can recognize L in real time.*
- *There is an automaton with von Neumann neighborhood which recognizes L in real time.*

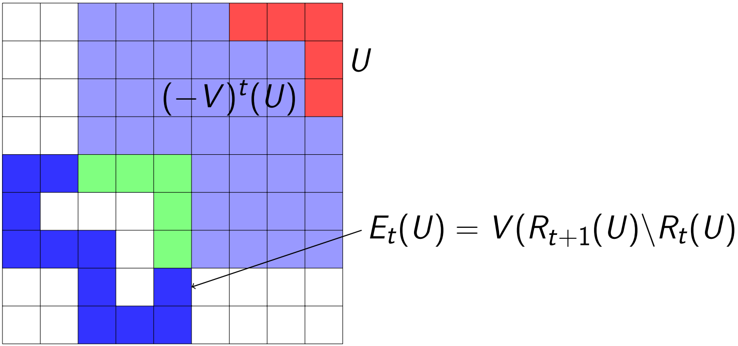
Language L



Notations



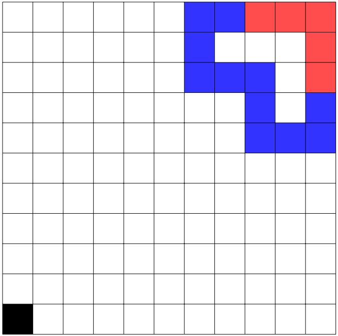
U – *equivalence*



Definition

C and C' are U – *equivalent* iff
For any configuration D on U, at any time t, the states of the cells in $V(R_{t+1}(U)) \setminus R_t(U)$ are the same on the input $C \oplus D$ and $C' \oplus D$.

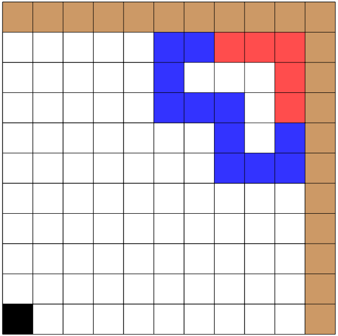
First lemma



Lemma

If two configurations C and C' are $U-equivalent$, then for any D on U, the output of the automaton is the same for $C \oplus D$ and $C' \oplus D$.

First lemma



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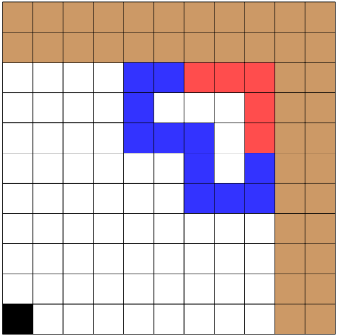

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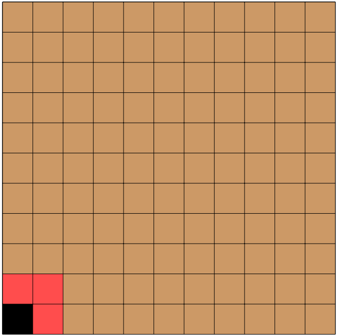

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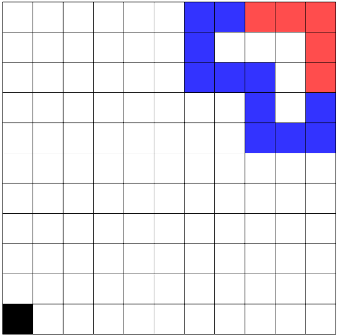
Second lemma



Lemma

There is at most $2^{O(n \log n)}$ $U-equivalence\ classes$.

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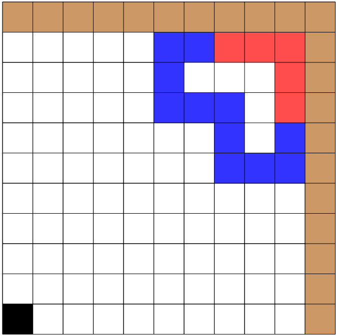

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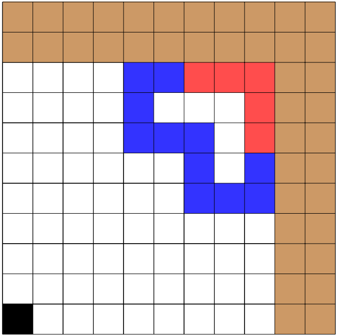

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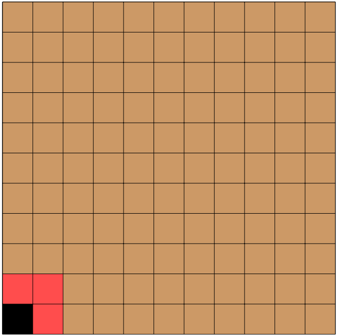

Lemma

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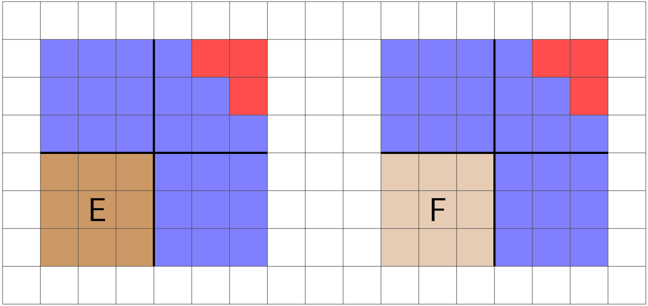
Contradiction and conclusion

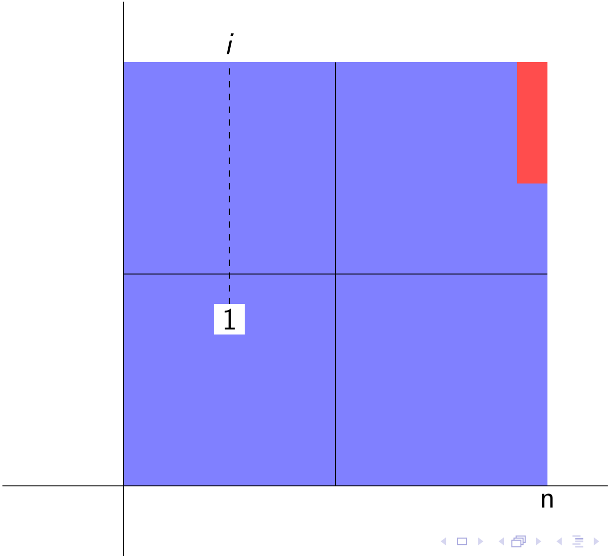


Lemma

There should be $2^{\Omega(n^2)}$ $U-$ *equivalence classes.*

Thus there is a contradiction, therefore no automaton with Moore neighborhood can recognize L in real time.

How to recognize L in real time with von Neumann neighborhood



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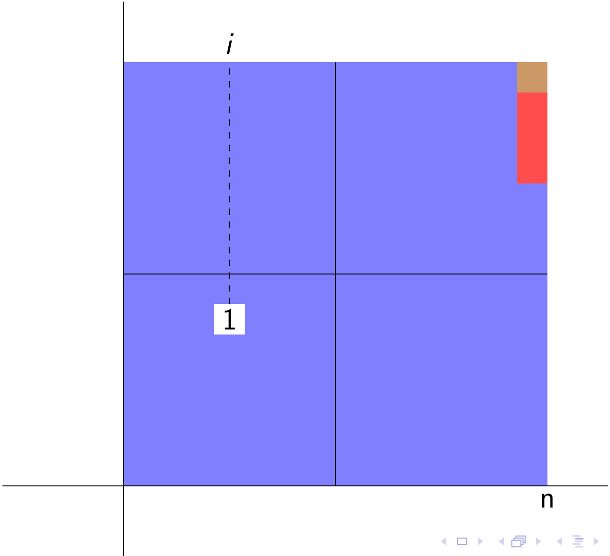

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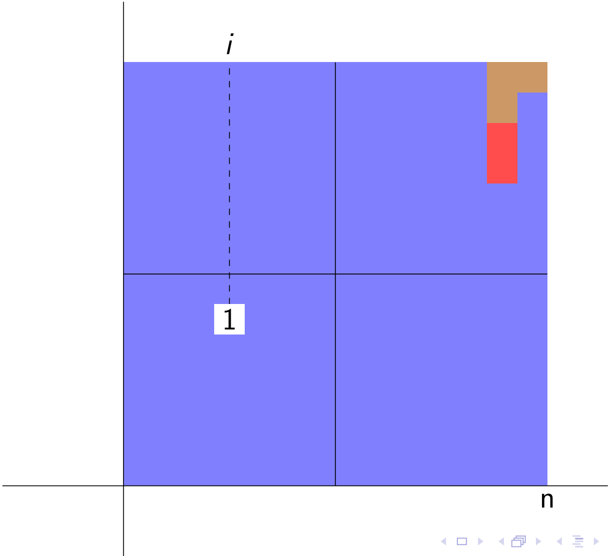

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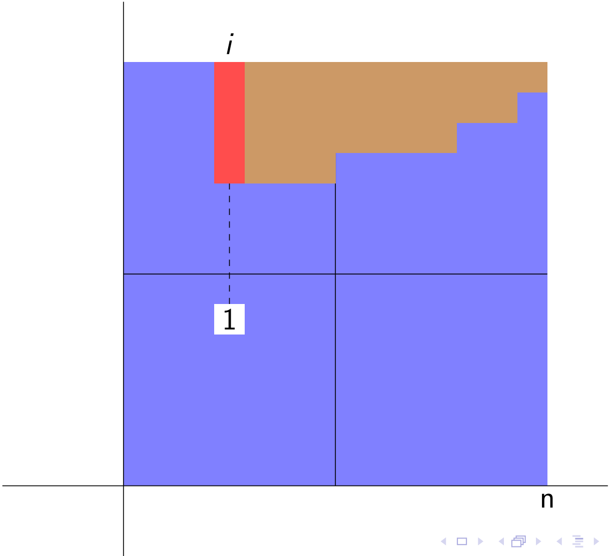

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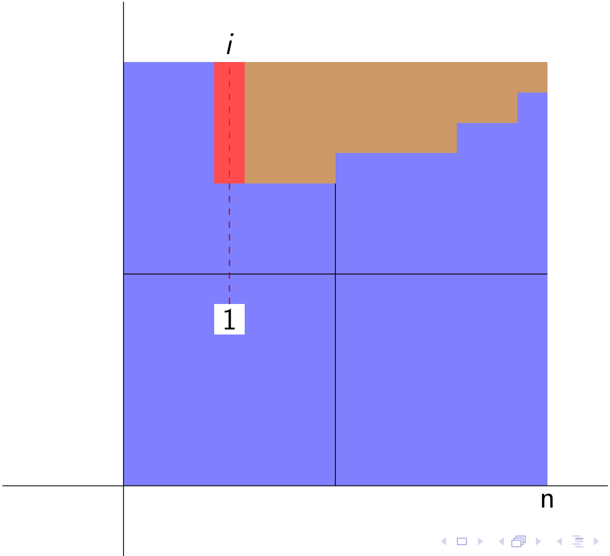

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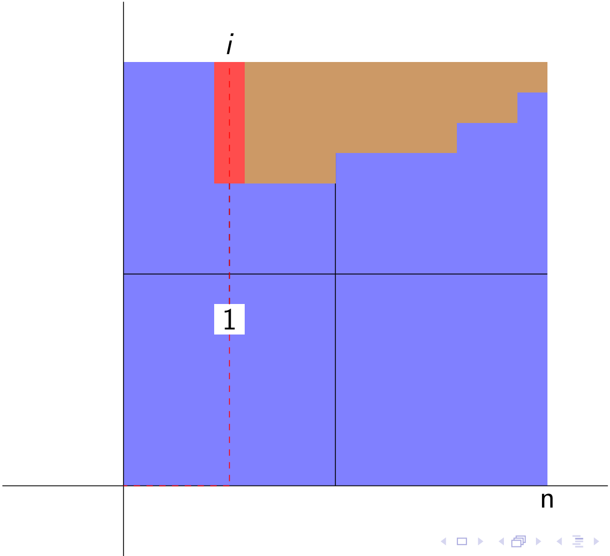
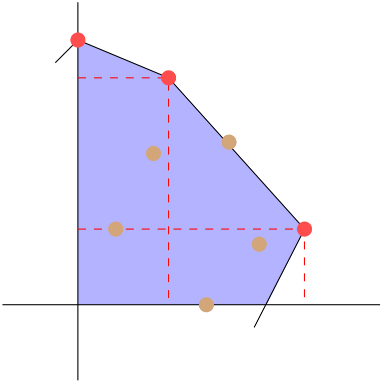


Limiting vertex

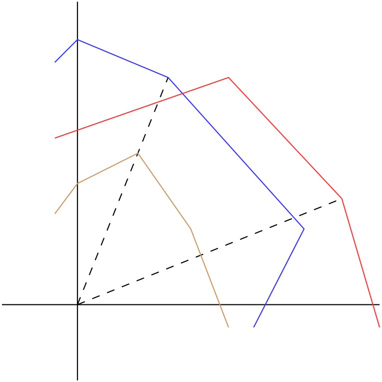


Definition

A vertex (x, y) of a convex hull of a neighborhood V is said to be *limiting* if :

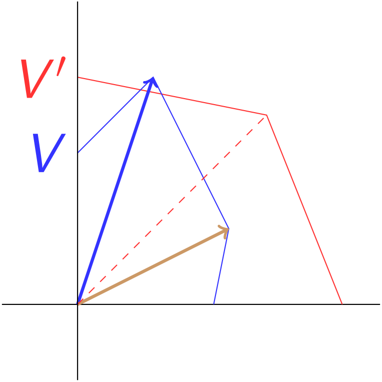
$$x, y > 0, [0, x] \times [0, y] \subset CCH(V)$$

A new result

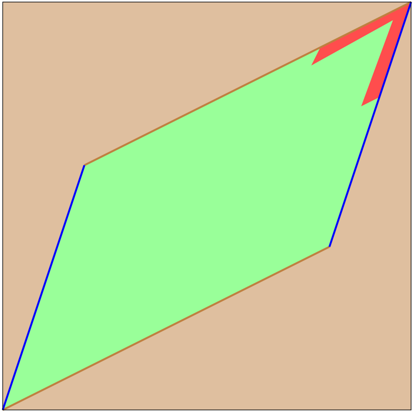


Proposition

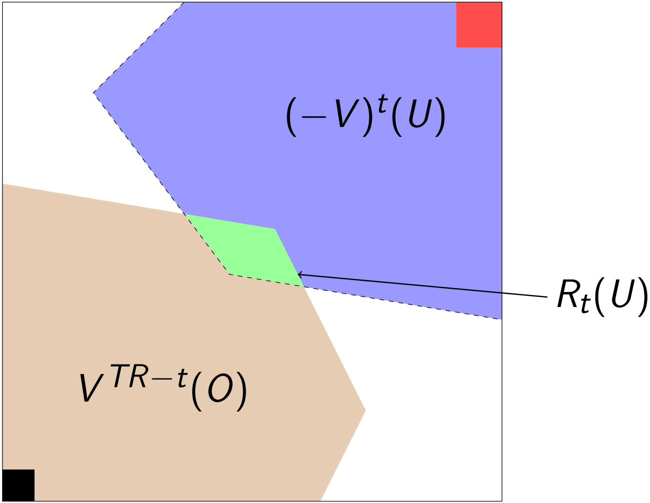
If V admits one limiting vertex, and V' does not have any colinear vertex, then real time is different for V and V'.


V′
V

The new language built for V'



but not for V



Conclusion

- If V have a limiting vertex, for all complete V' which does not have a linear vertex : $RT(V') \not\subseteq RT(V)$.
- If V and V' have both one limiting vertex the other does not have $RT(V) \neq RT(V')$
- If V has a limiting vertex, then real time and linear time are different for V.