Field-Based Firing Squad Solution for a Large Class of Cellular Spaces

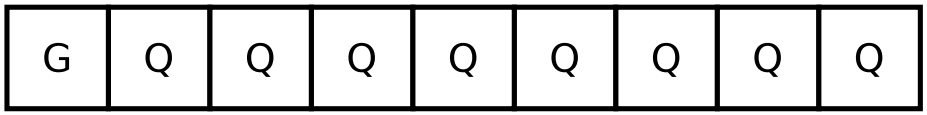
Luidnel Maignan[1], Jean-Baptiste Yunès[2]
[1] LACL, Université Paris-Est Créteil
[2] LIAFA, Université Paris Diderot, France

FRAC d'Automne 2013, Orléans, France
October 25, 2013

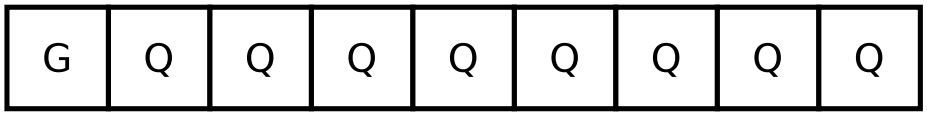
Introduction

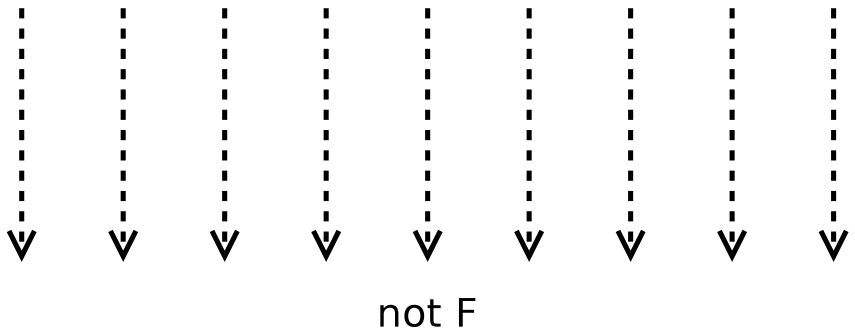
- Cellular automata
- FSSP Movations (self reproduction)
- Signals versus Fields

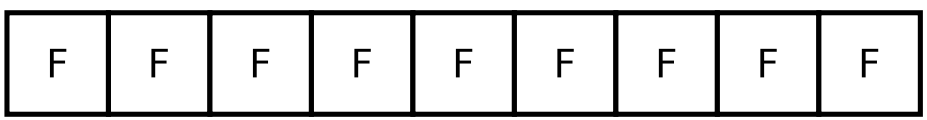
Firing Squad Synchronization Problems (1D)



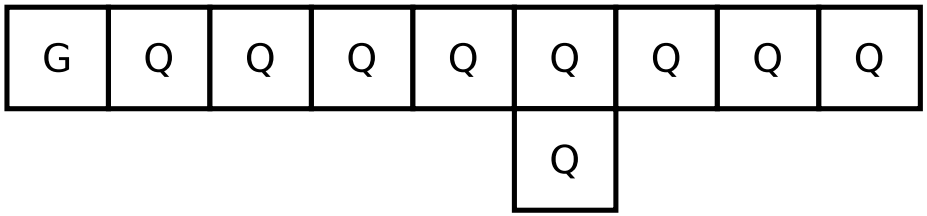
Firing Squad Synchronization Problems (1D)

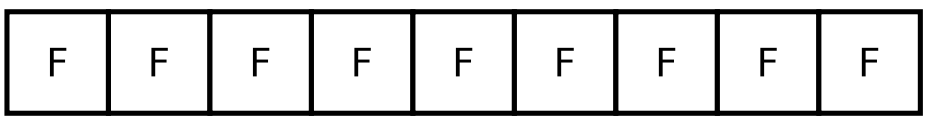




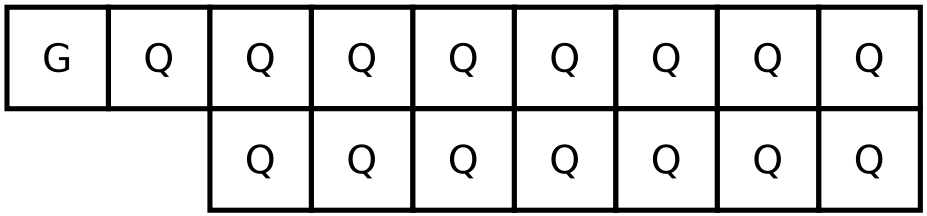


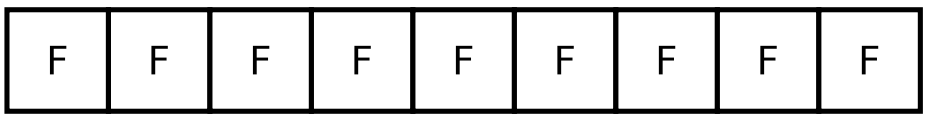
Firing Squad Synchronization Problems (1D)



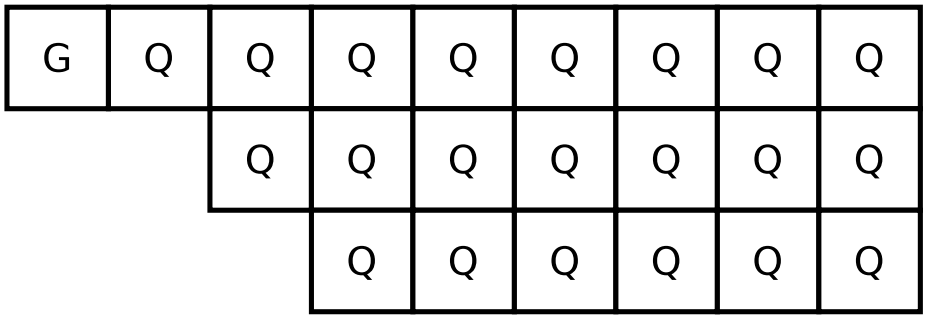


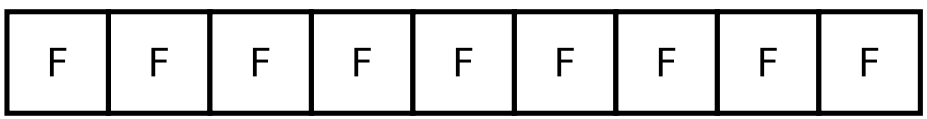
Firing Squad Synchronization Problems (1D)



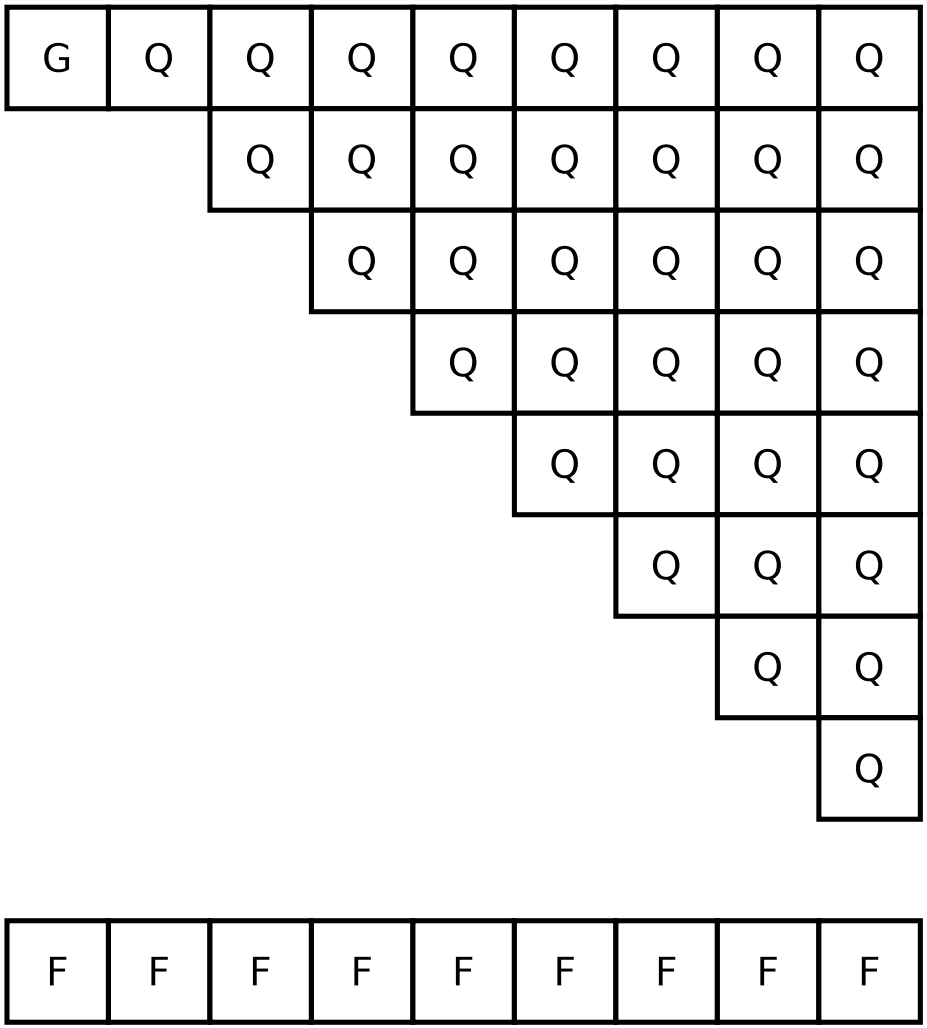


Firing Squad Synchronization Problems (1D)

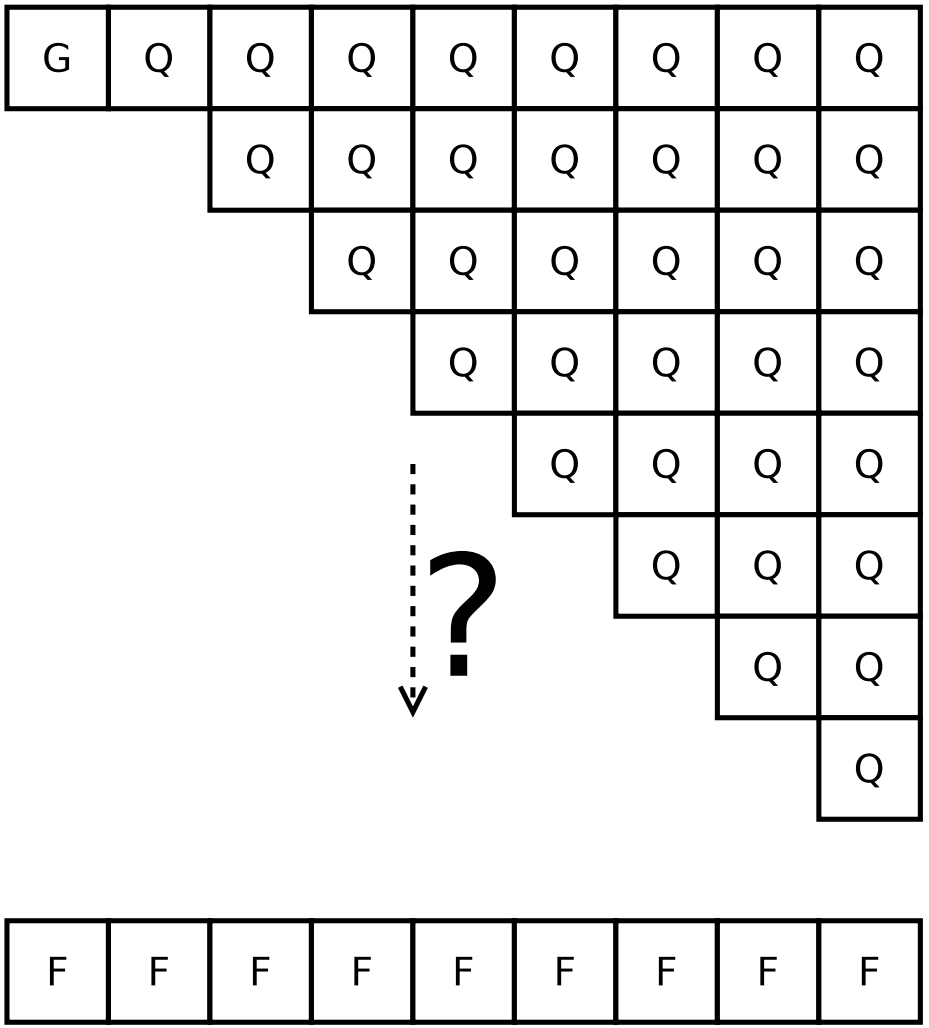




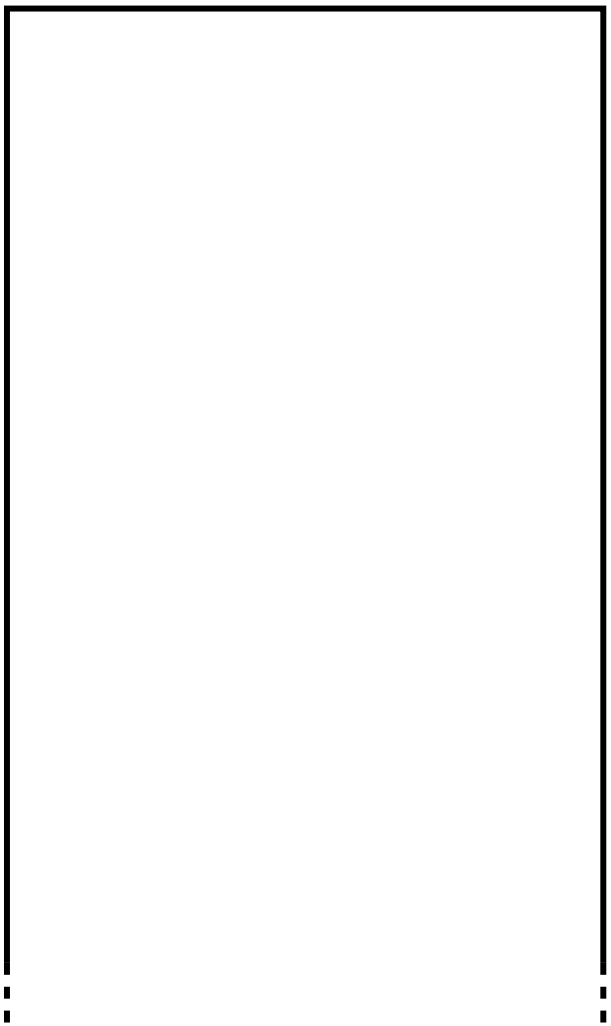
Firing Squad Synchronization Problems (1D)



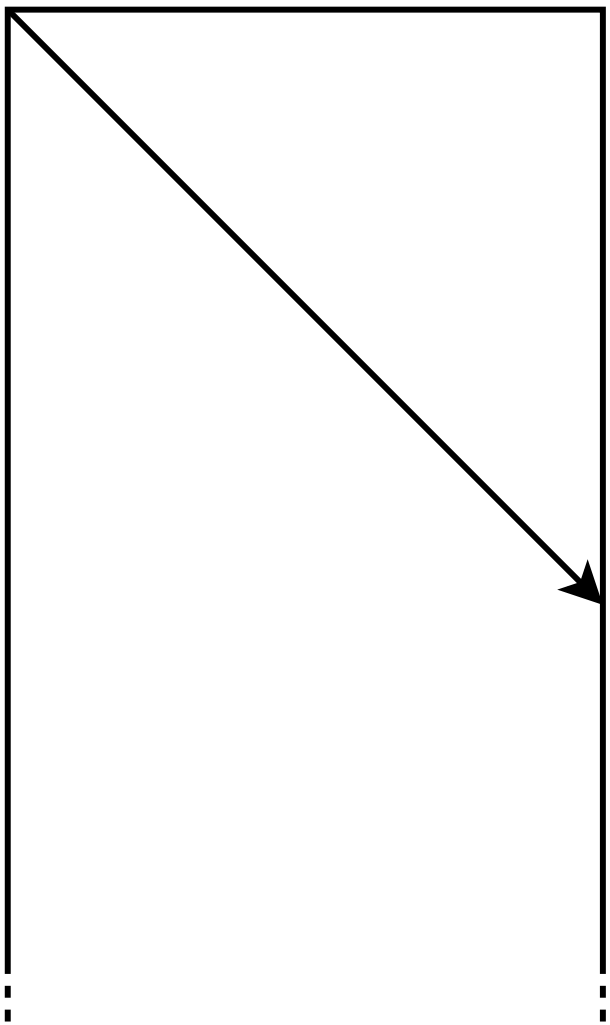
Firing Squad Synchronization Problems (1D)



Signals, Speeds and Collisions

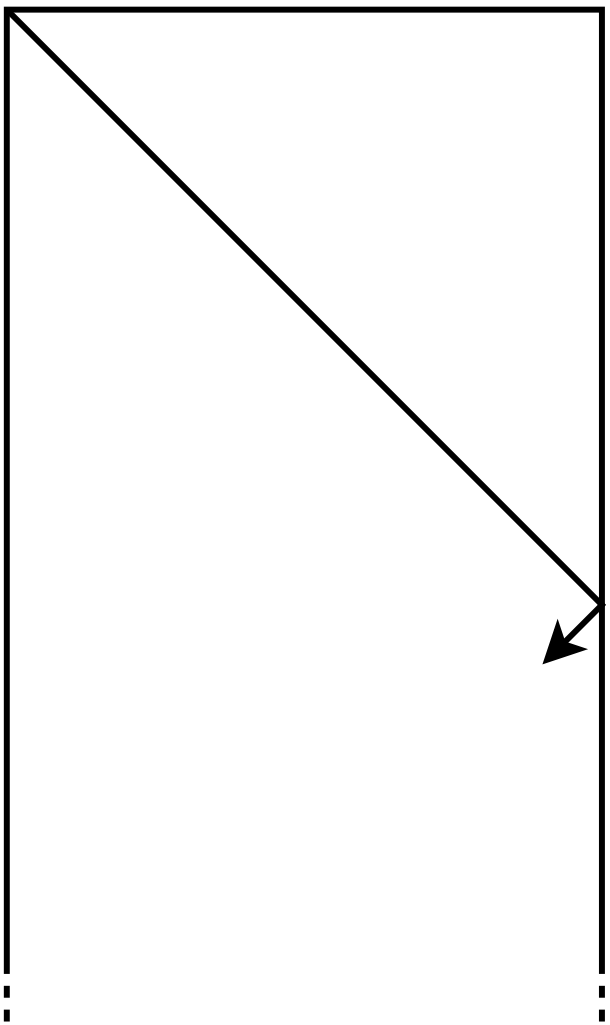


Signals, Speeds and Collisions



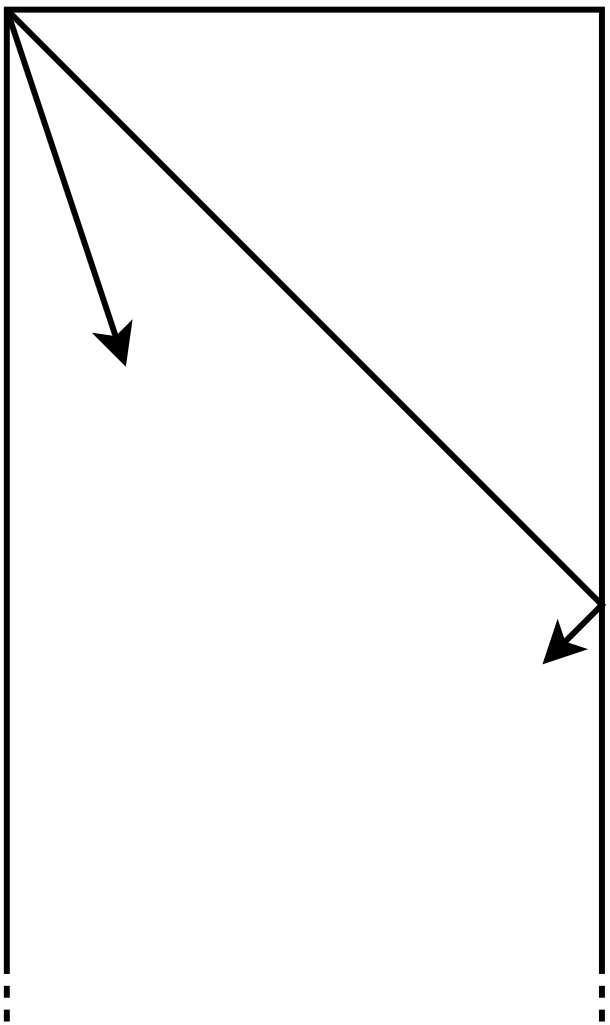
- Signal of speed 1...

Signals, Speeds and Collisions



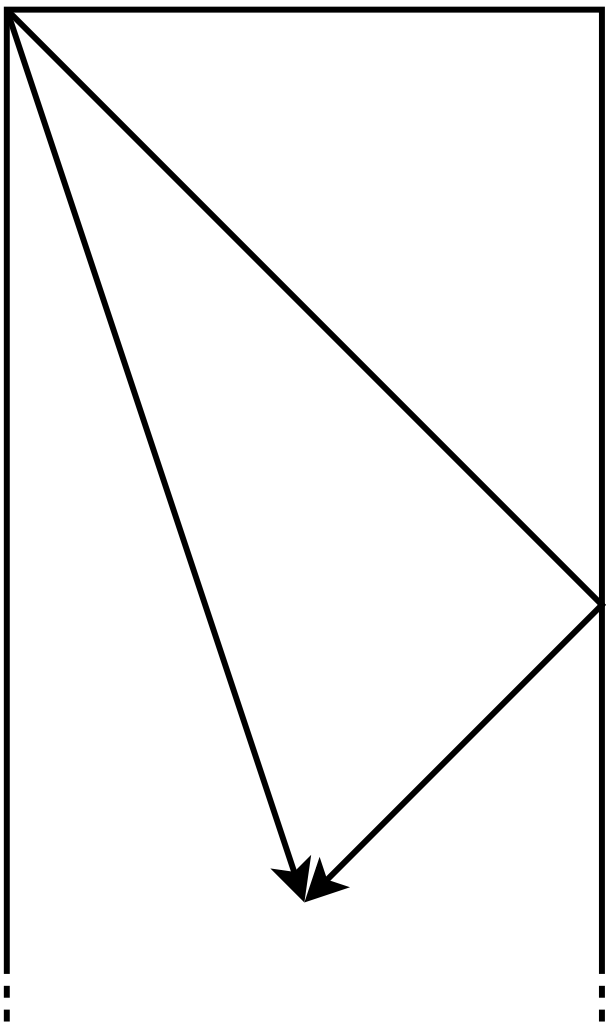
- Signal of speed 1...
- ...bouncing on the border

Signals, Speeds and Collisions



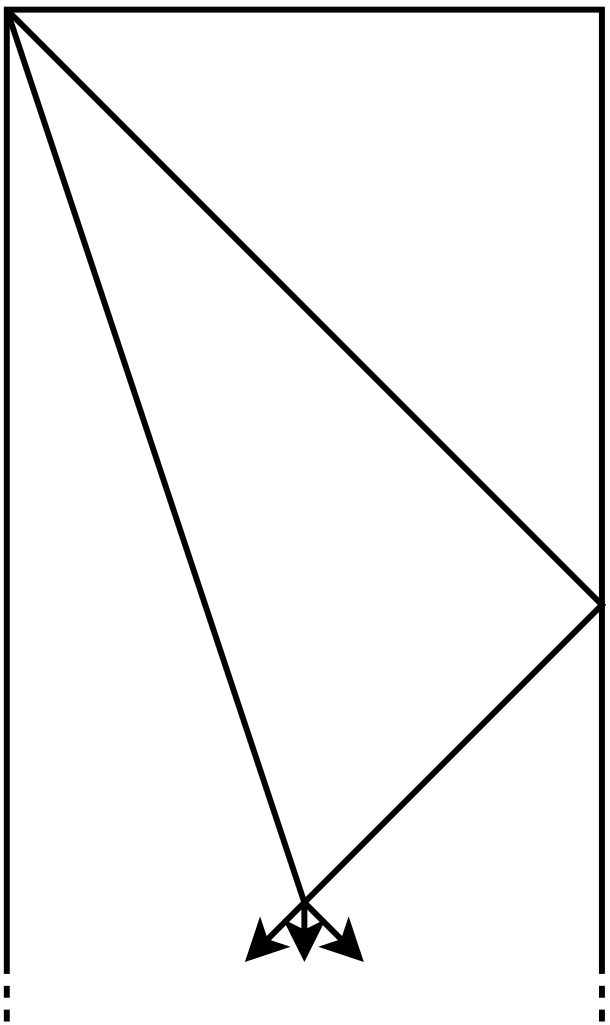
- Signal of speed 1...
- ...bouncing on the border
- Signal of speed $\frac{1}{3}$

Signals, Speeds and Collisions

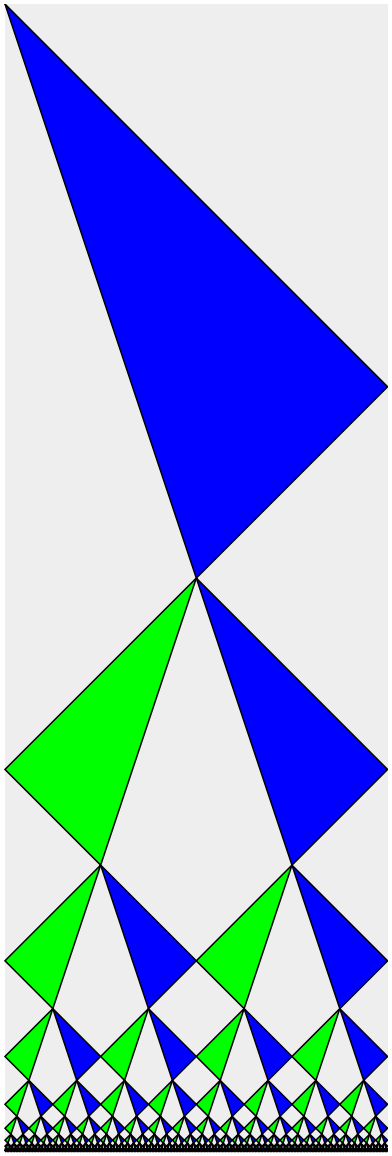


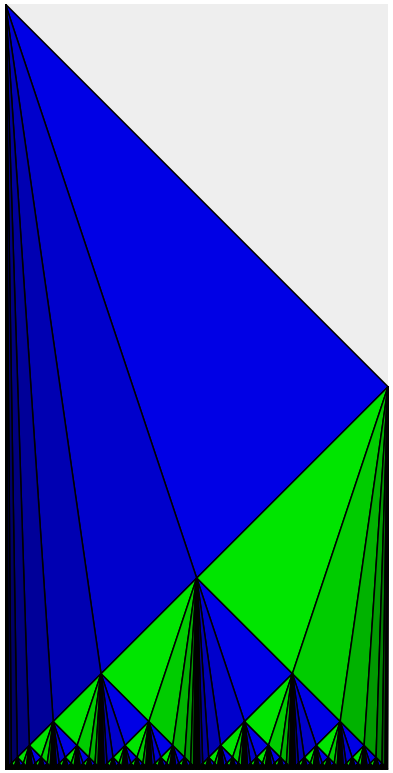
- Signal of speed 1...
- ...bouncing on the border
- Signal of speed $\frac{1}{3}$
- $\Rightarrow$ Collision at the middle

Signals, Speeds and Collisions



- Signal of speed 1...
- ...bouncing on the border
- Signal of speed $\frac{1}{3}$
- $\Rightarrow$ Collision at the middle
- Creation of new signals...





Modularity using fields

Examples:

- Traffic simulator
- Asynchronous Multi-General FSSP

A Field-Based Description

What we want:

- Take the whole space

A Field-Based Description

What we want:

- Take the whole space
- Cut it in 2 equal regions

A Field-Based Description

What we want:

- Take the whole space
- Cut it in 2 equal regions
- Cut each in 2 equal regions

A Field-Based Description

What we want:

- Take the whole space
- Cut it in 2 equal regions
- Cut each in 2 equal regions
- Repeat as long as you can

A Field-Based Description

What we want:

- Take the whole space
- Cut it in 2 equal regions
- Cut each in 2 equal regions
- Repeat as long as you can
- Fire

A Field-Based Description

What we want:

- Take the whole space
- Cut it in 2 equal regions
- Cut each in 2 equal regions
- Repeat as long as you can
- Fire

What we need:

- Description of regions
- Computation of middles

A Field-Based Description

What we want:

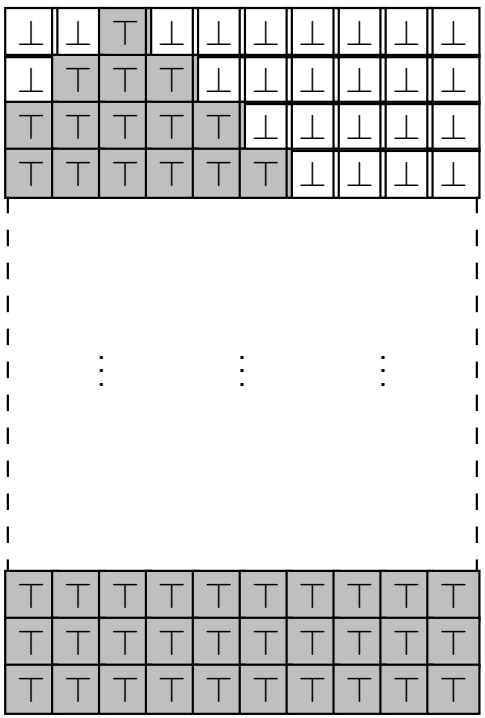
- Take the whole space
- Cut it in 2 equal regions
- Cut each in 2 equal regions
- Repeat as long as you can
- Fire

What we need:

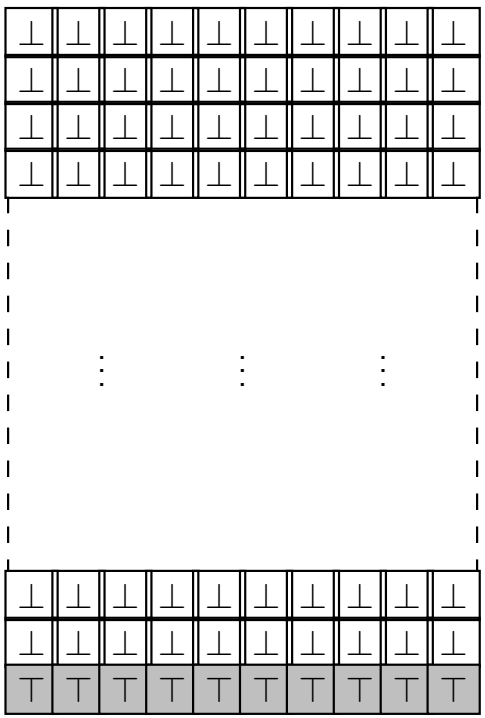
- Description of regions
- Computation of middles

How we do:

- Region fields
- Distance fields



(a) input field



(b) output field

Initial region

$$\mathrm{brd}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \exists i \in \mathrm{I};\ c + i \notin \mathrm{S},$$
$$\mathrm{ins}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \forall i \in \mathrm{I};\ c + i \in \mathrm{S},$$

$$\mathrm{I} = \{-1, +1\}$$

Demo 1

Middle of the initial region

$$\mathrm{dst}^{\ell}_{t+1}(c) = \begin{cases} 1 + \min_{i \in \mathrm{I}}\{\mathrm{dst}^{\ell}_{t}(c+i)\} & \text{if } \mathrm{ins}^{\ell}_{t+1}(c) \\ 0 & \text{otherwise.} \end{cases}$$

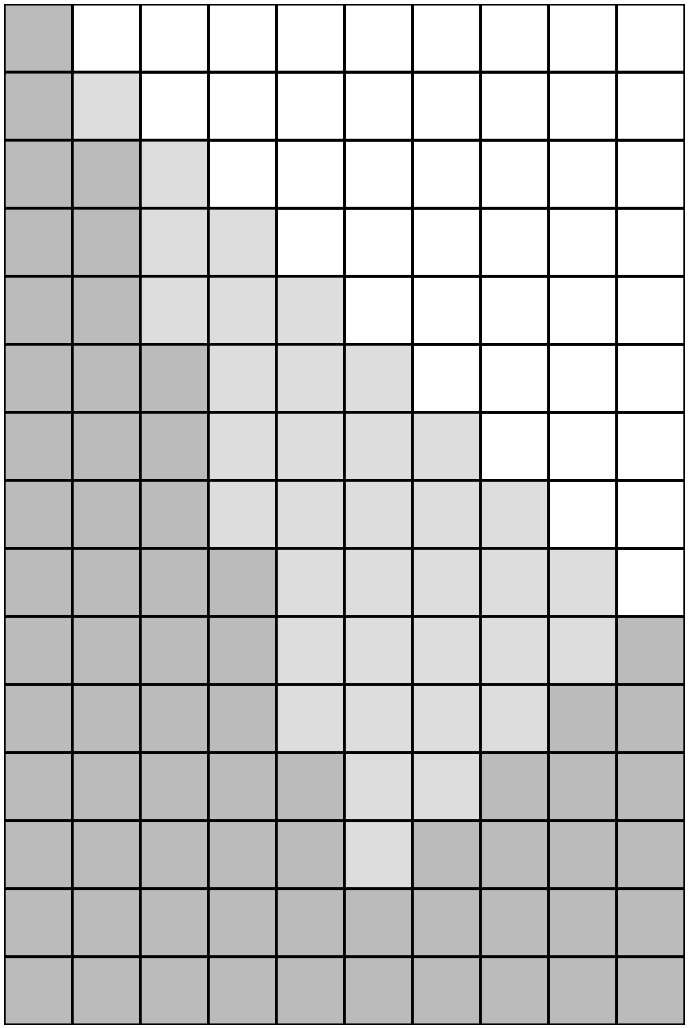
Demo 2

Stability and correctness of the initial division

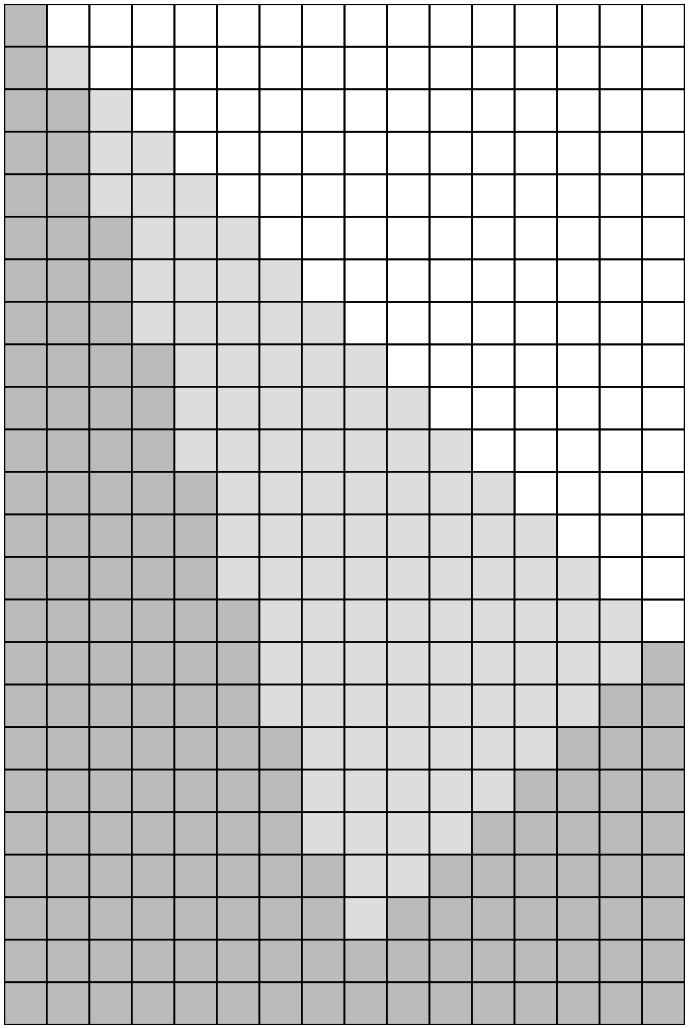
$$\mathrm{sta}^{\ell}_{t+1}(c) = \bigvee \begin{cases} \mathrm{brd}^{\ell}_{t+1}(c) \\ \exists i \in \mathrm{I};\ \mathrm{dst}^{\ell}_{t+1}(c) = 1 + \mathrm{dst}^{\ell}_{t}(c+i) \wedge\ \mathrm{sta}^{\ell}_{t}(c+i). \end{cases}$$

Demo 2

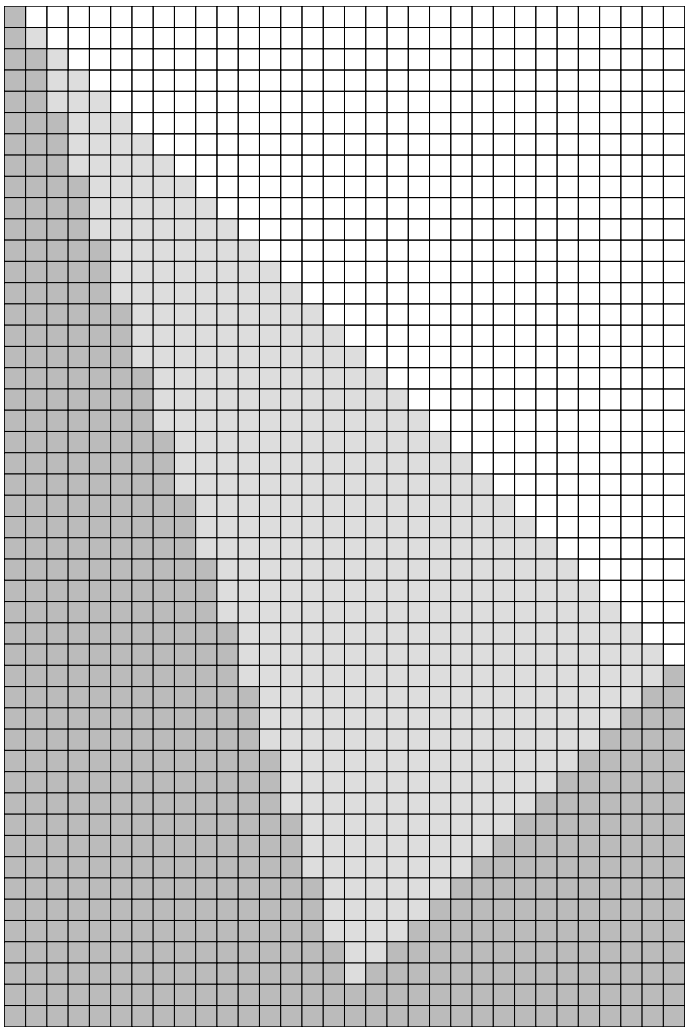
Space-time diagram of the level 0



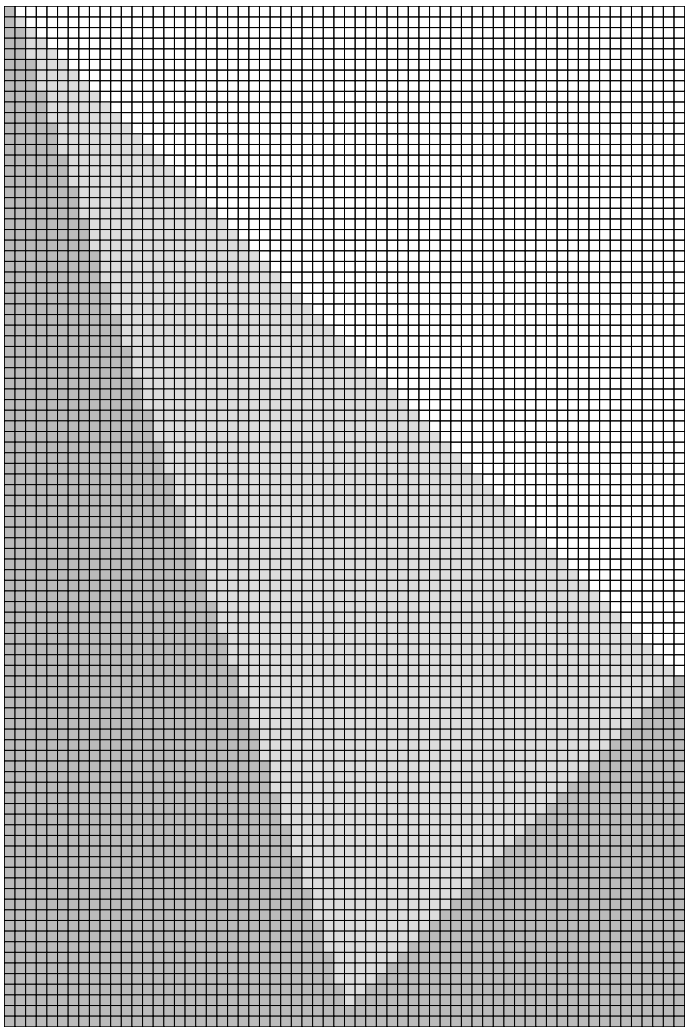
Space-time diagram of the level 0



Space-time diagram of the level 0



Space-time diagram of the level 0



Add another level of division

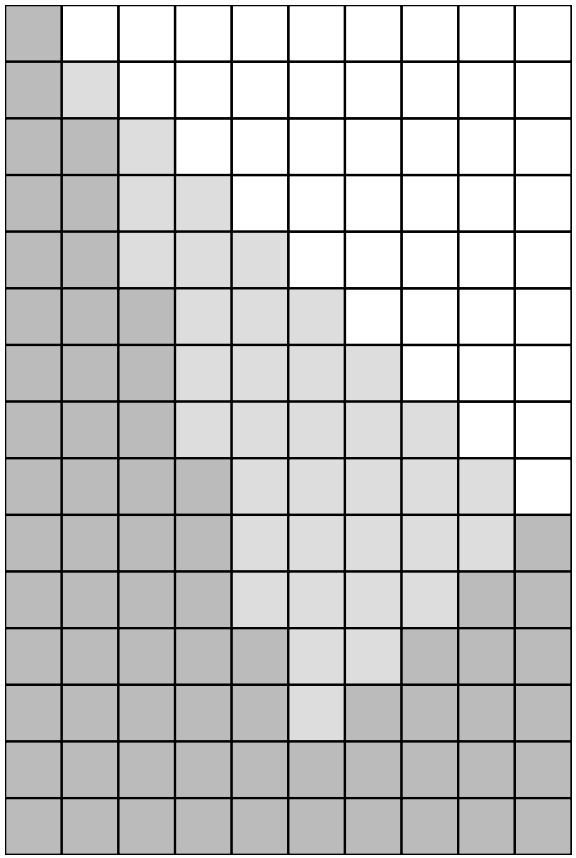
- dst^0 and sta^0 computed from brd^0 and ins^0
- half-space region informations: brd^1 and ins^1
- Computed from level 0 informations: brd^0, ins^0, dst^0, sta^0

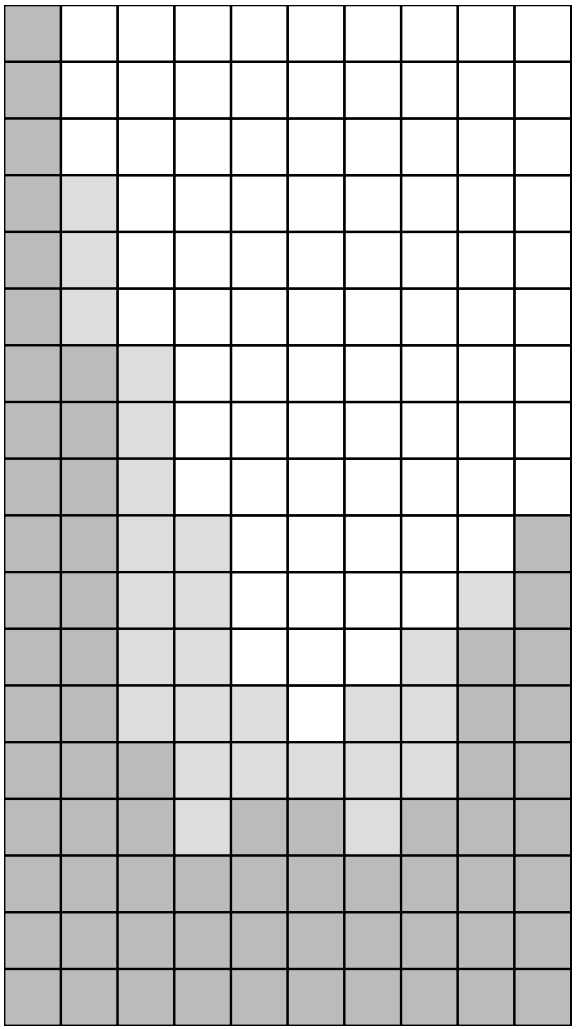
$\mathrm{brd}^1_{t+1}(c) =$ sure to be border or local maximum of level 0

$\mathrm{ins}^1_{t+1}(c) =$ sure to be inside and non local maximum at level 0,

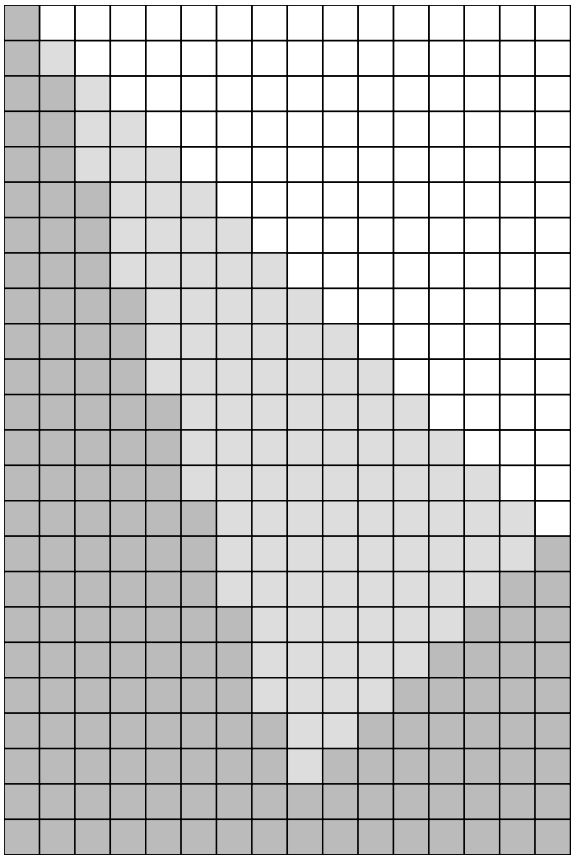
Demo 2

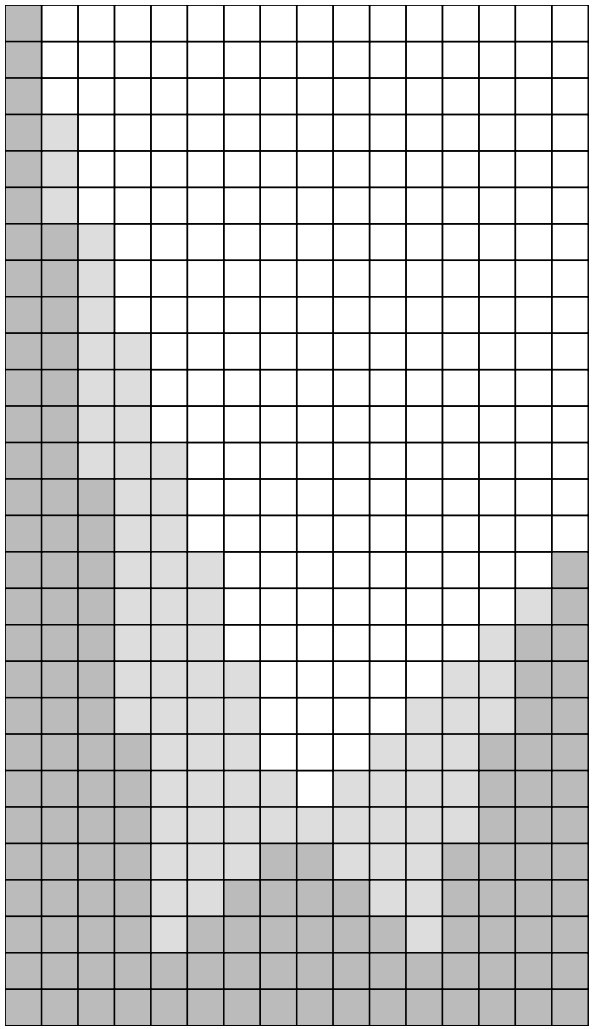
Space-time diagram of the level 1



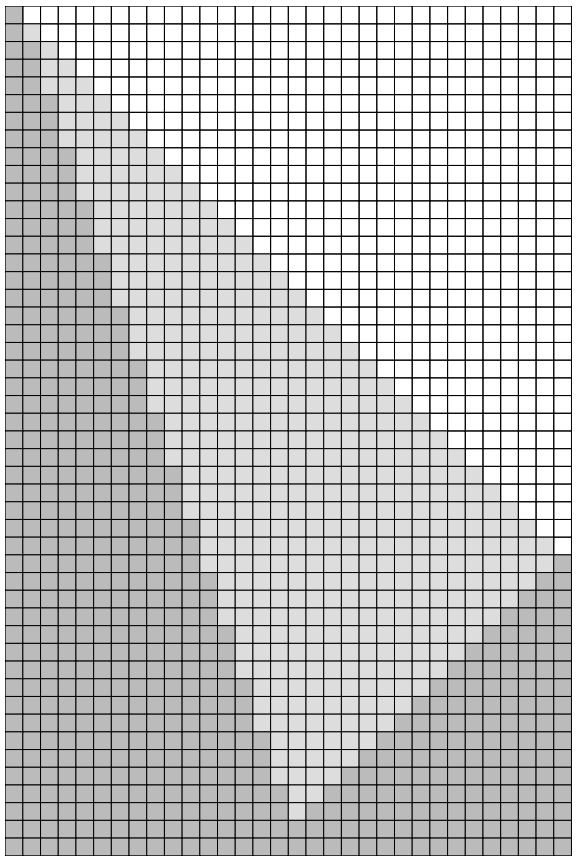


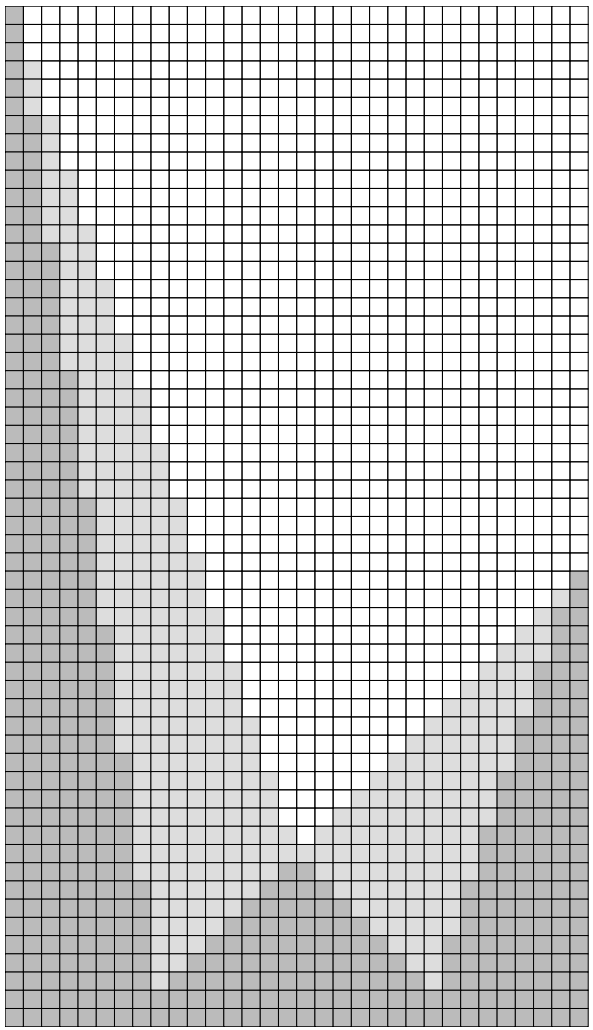
Space-time diagram of the level 1



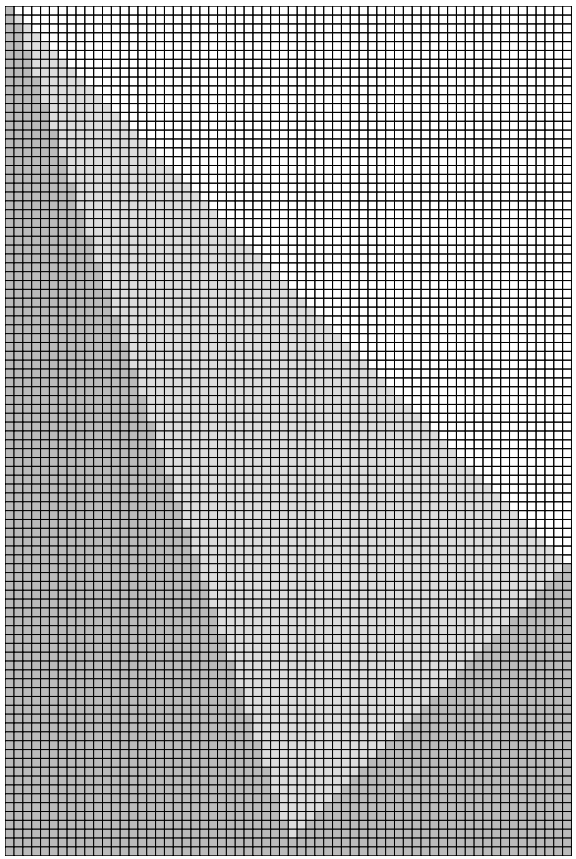


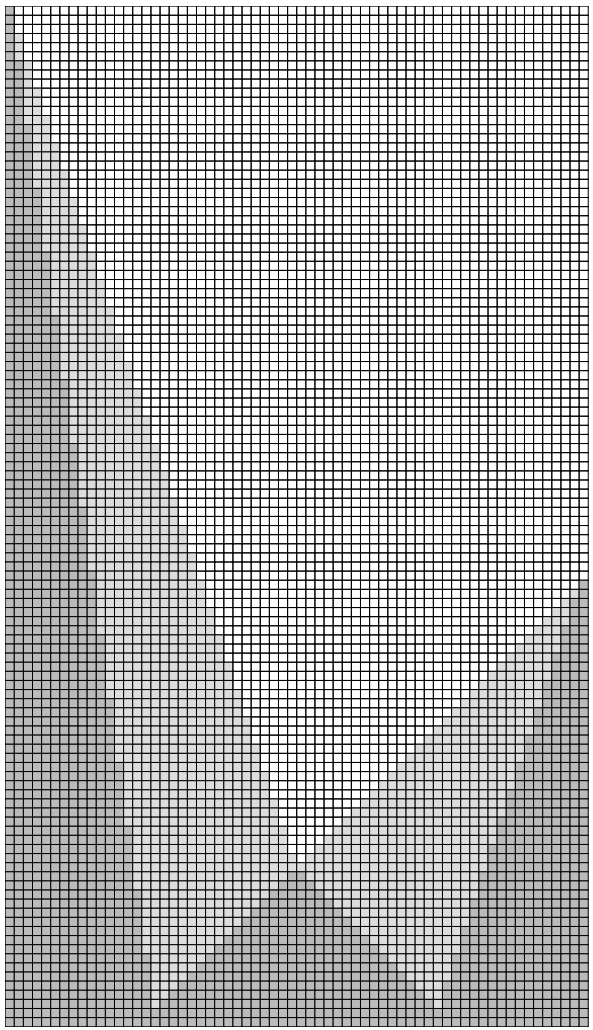
Space-time diagram of the level 1



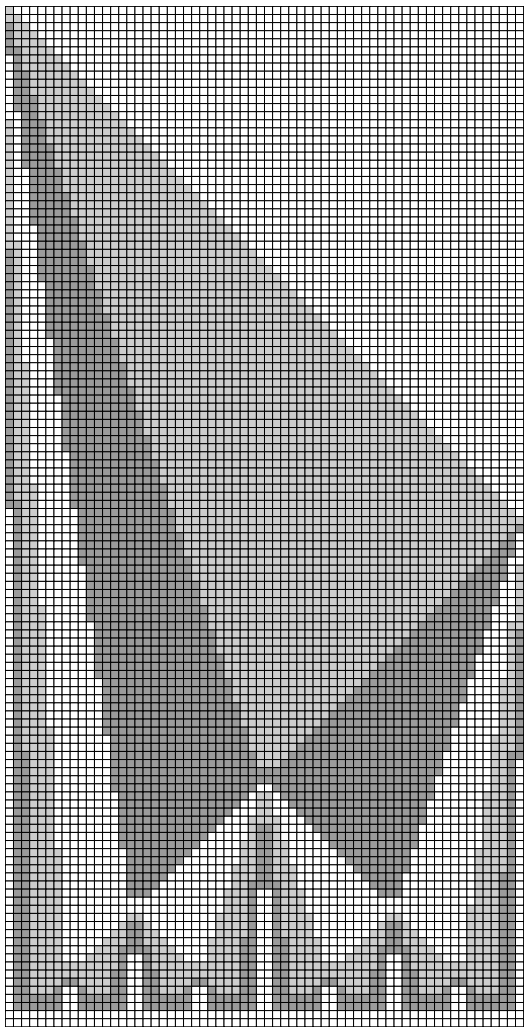


Space-time diagram of the level 1

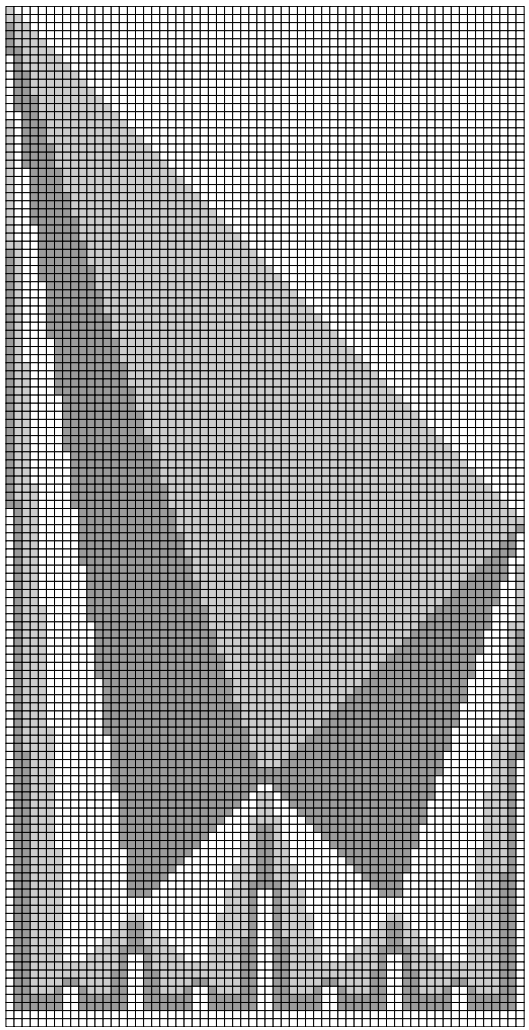


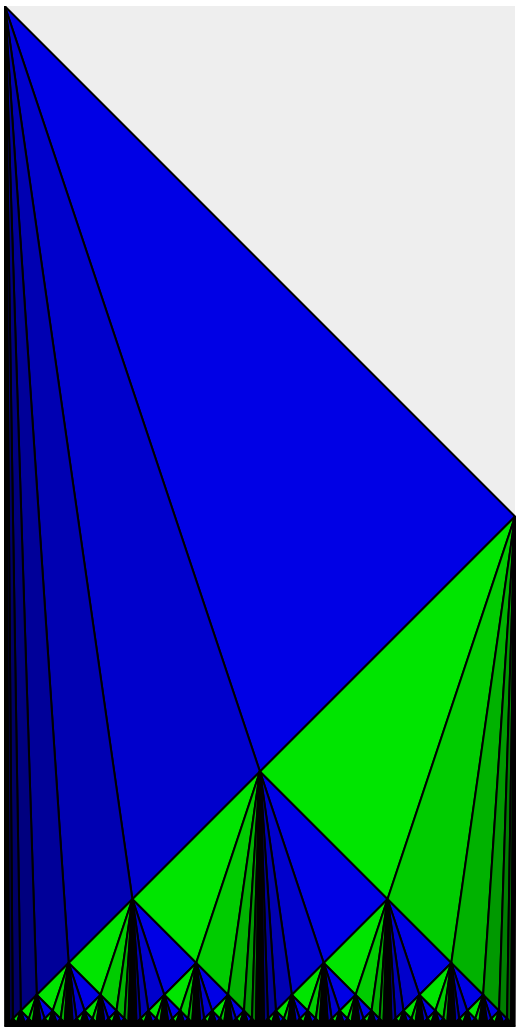


Space-time diagram of all levels

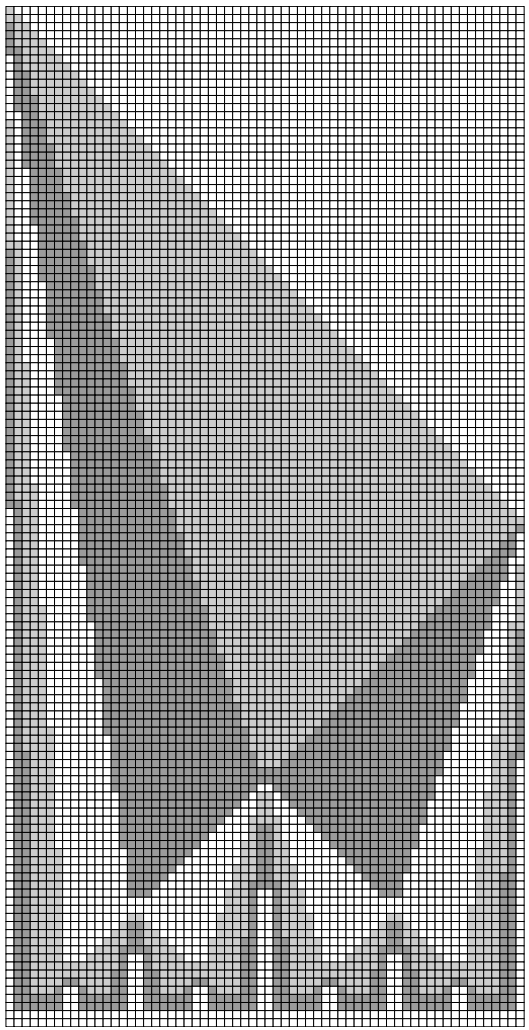


Space-time diagram of all levels

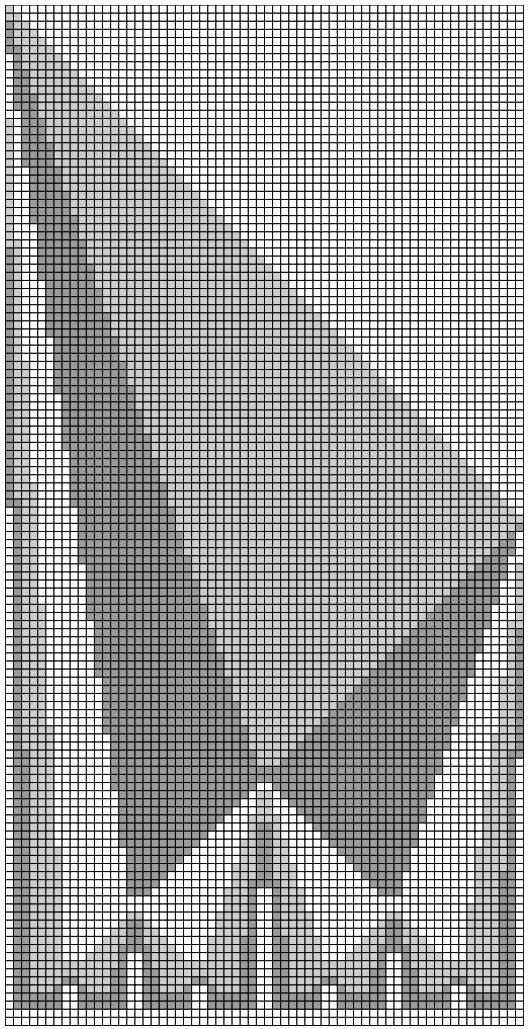




Space-time diagram of all levels

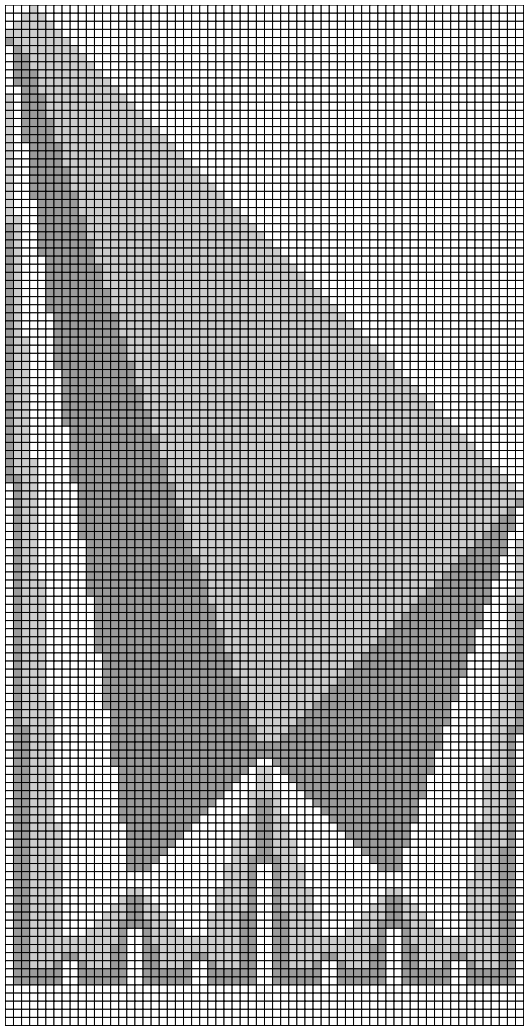


Space-Time Diagrams for different General's positions



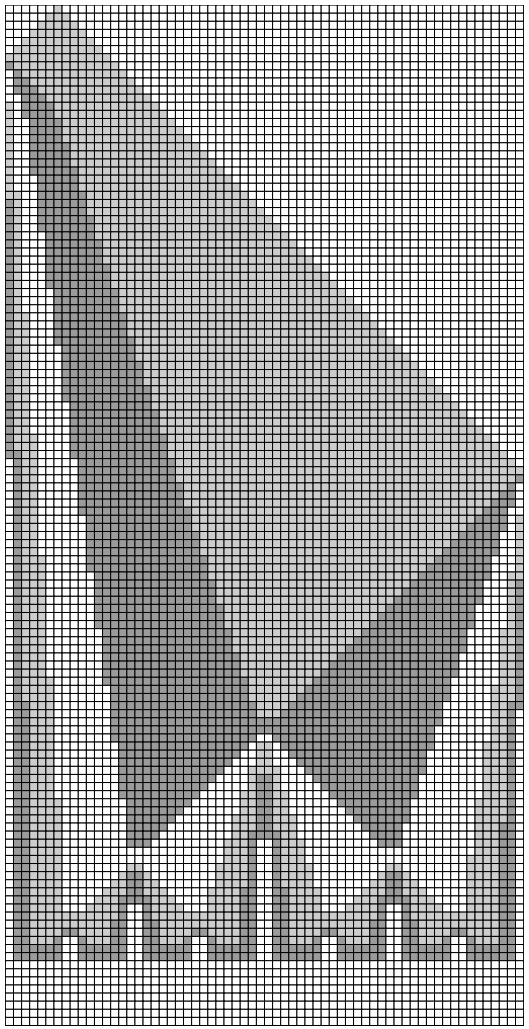
Demo 3

Space-Time Diagrams for different General's positions



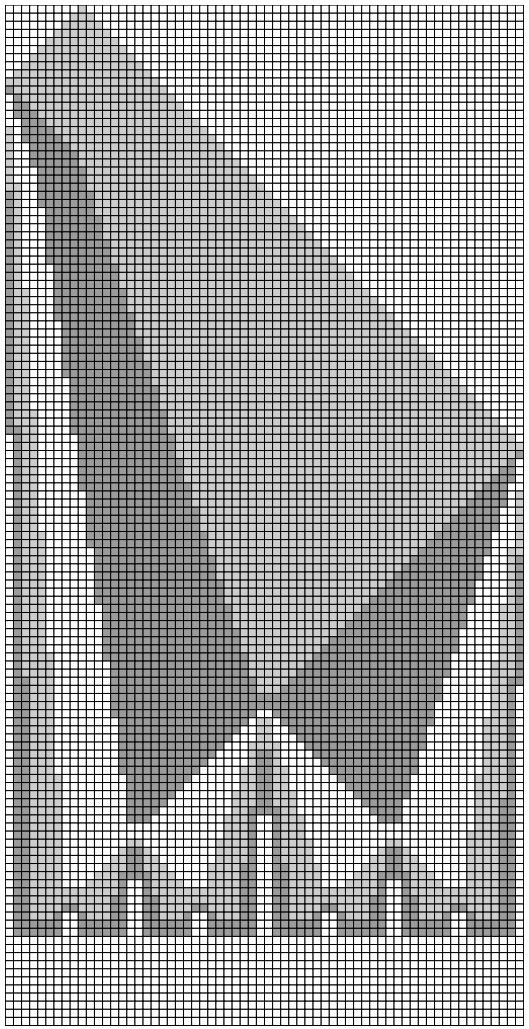
Demo 3

Space-Time Diagrams for different General's positions



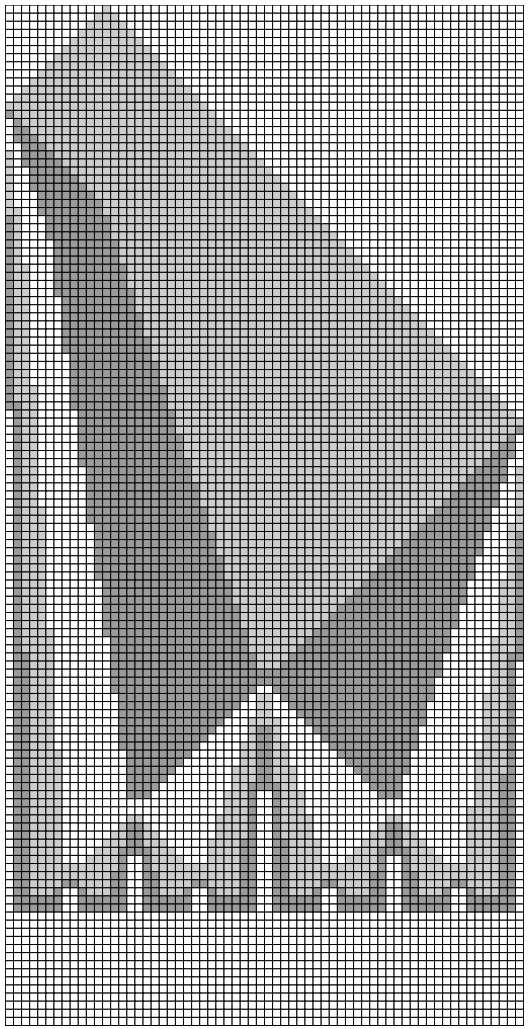
Demo 3

Space-Time Diagrams for different General's positions



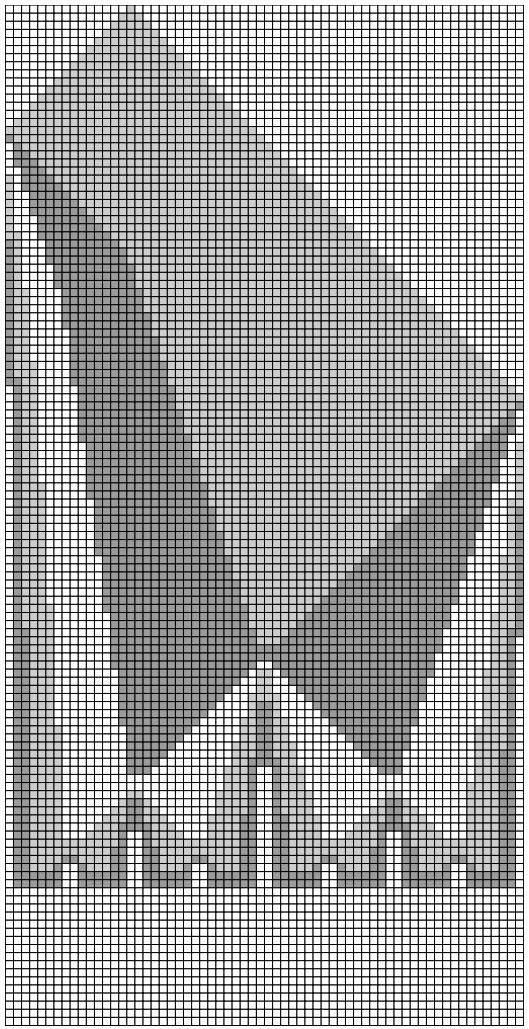
Demo 3

Space-Time Diagrams for different General's positions



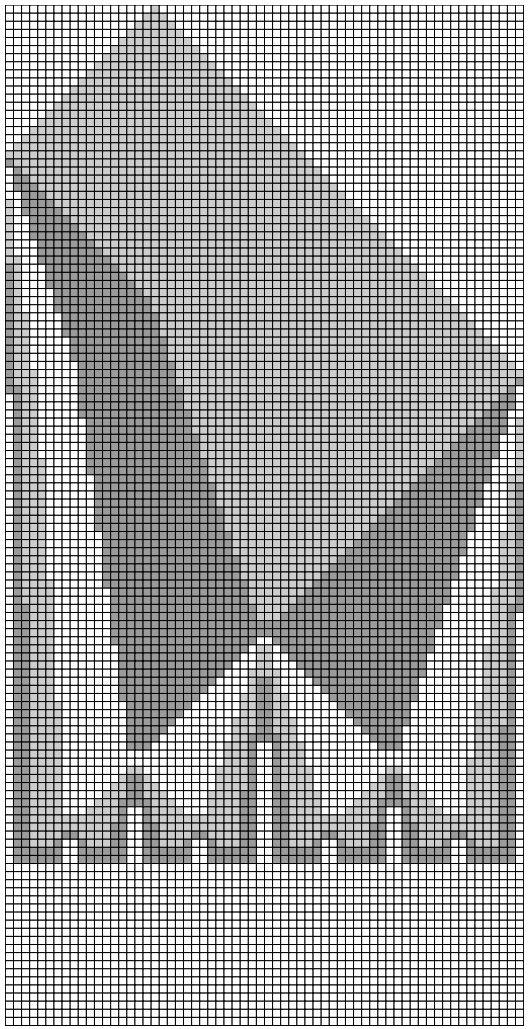
Demo 3

Space-Time Diagrams for different General's positions



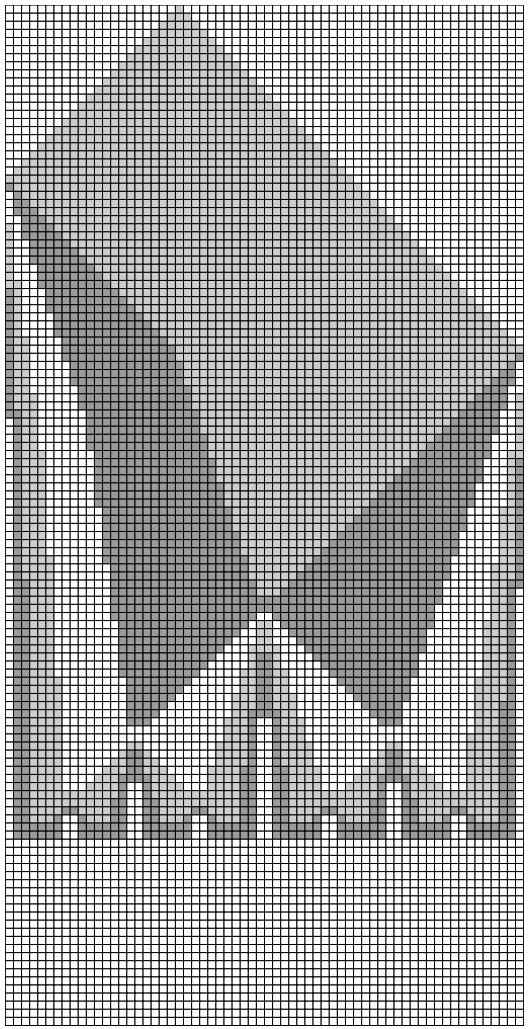
Demo 3

Space-Time Diagrams for different General's positions



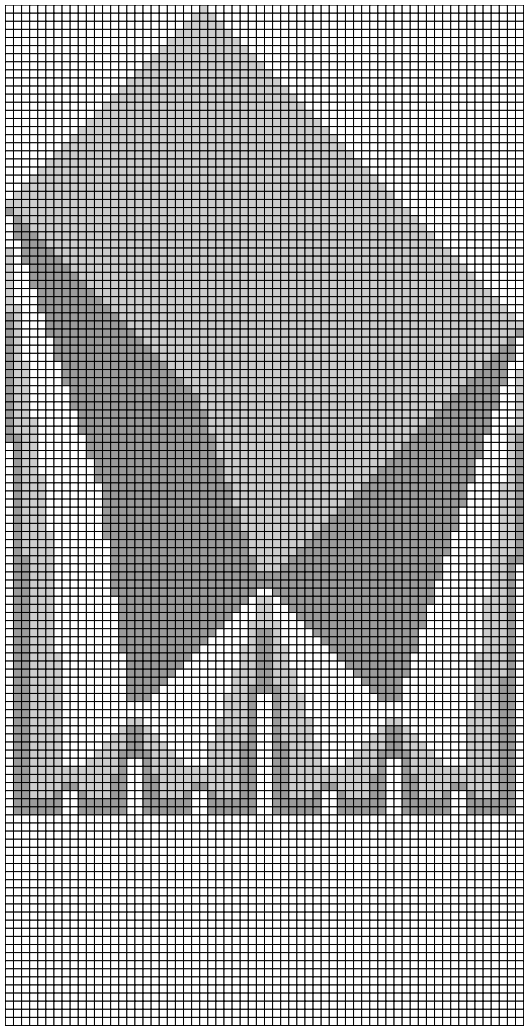
Demo 3

Space-Time Diagrams for different General's positions



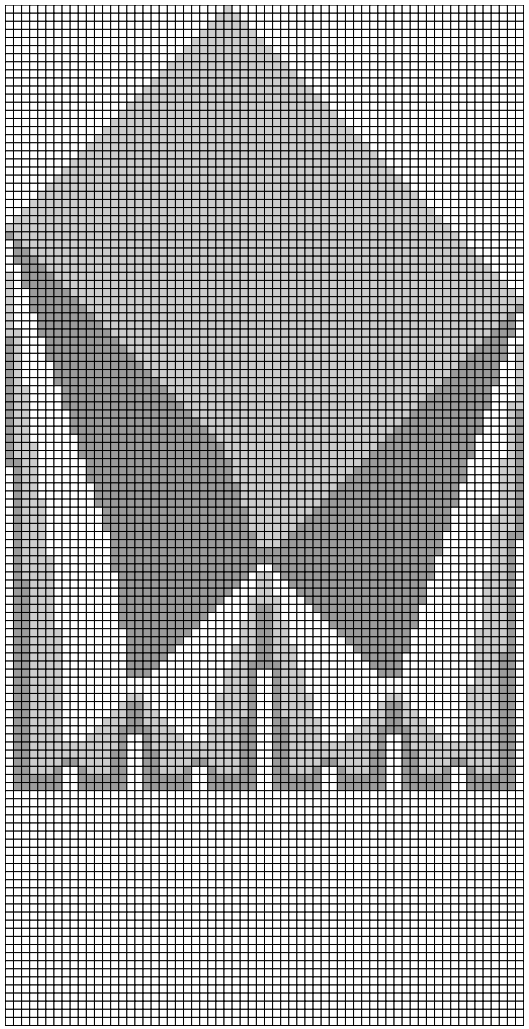
Demo 3

Space-Time Diagrams for different General's positions



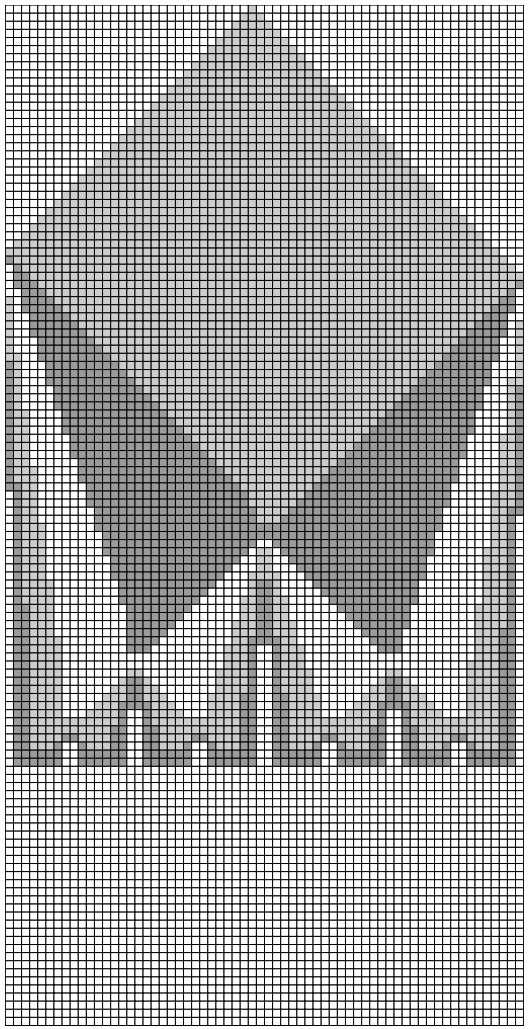
Demo 3

Space-Time Diagrams for different General's positions



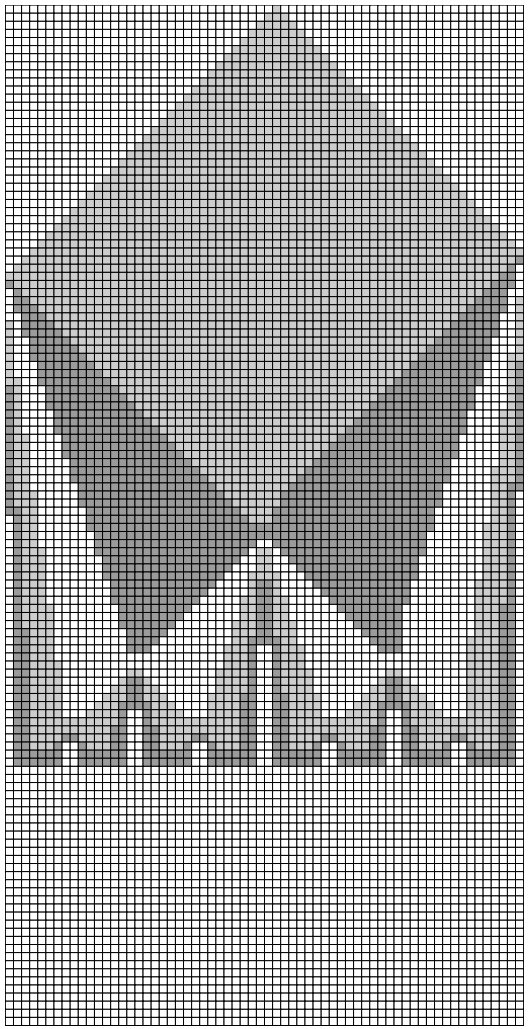
Demo 3

Space-Time Diagrams for different General's positions



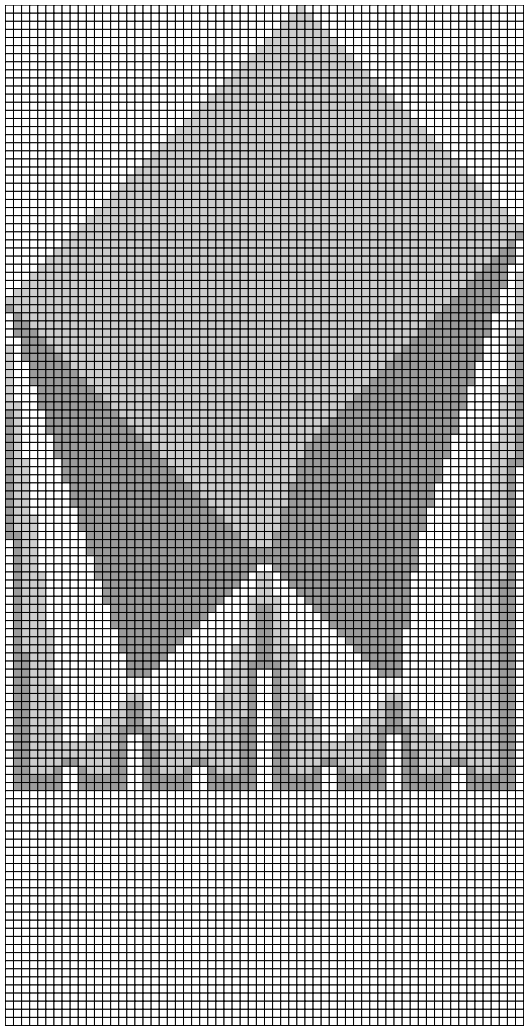
Demo 3

Space-Time Diagrams for different General's positions



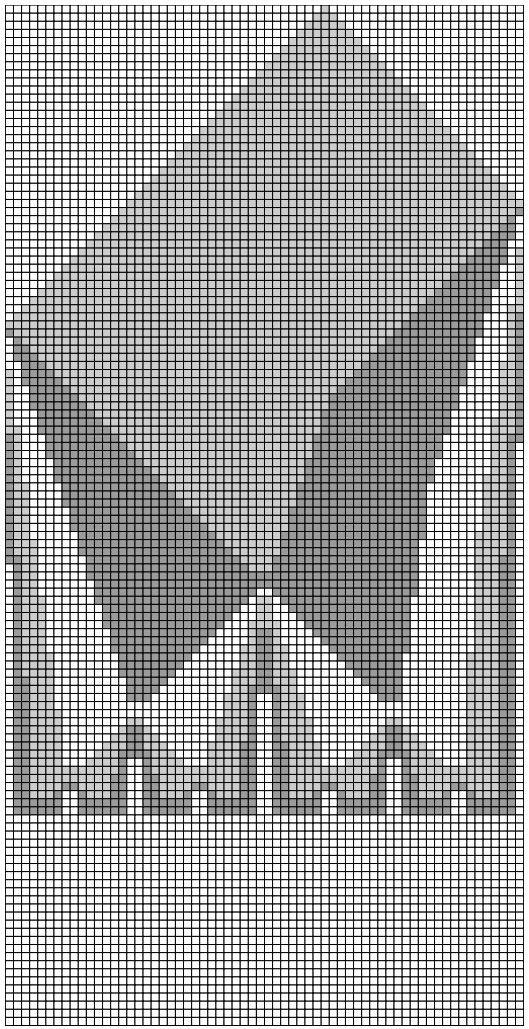
Demo 3

Space-Time Diagrams for different General's positions



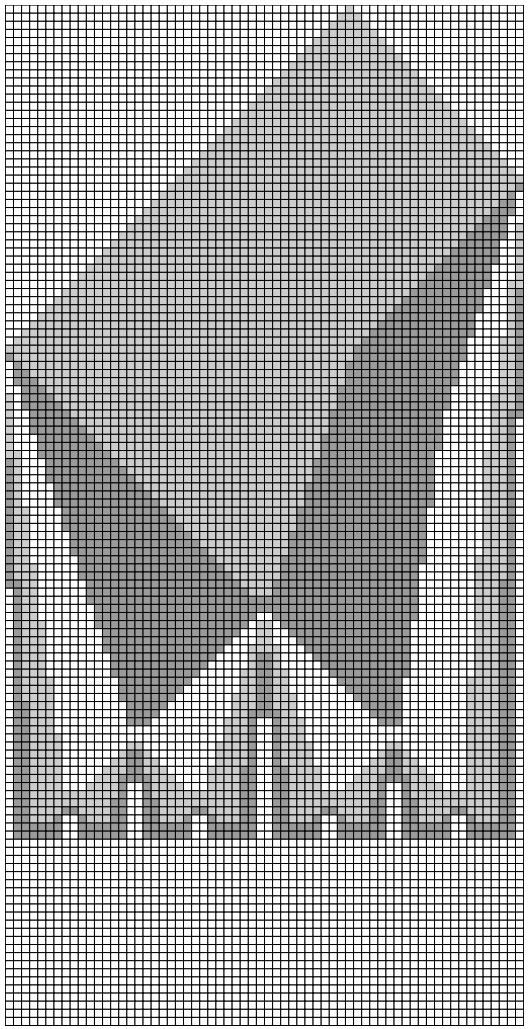
Demo 3

Space-Time Diagrams for different General's positions



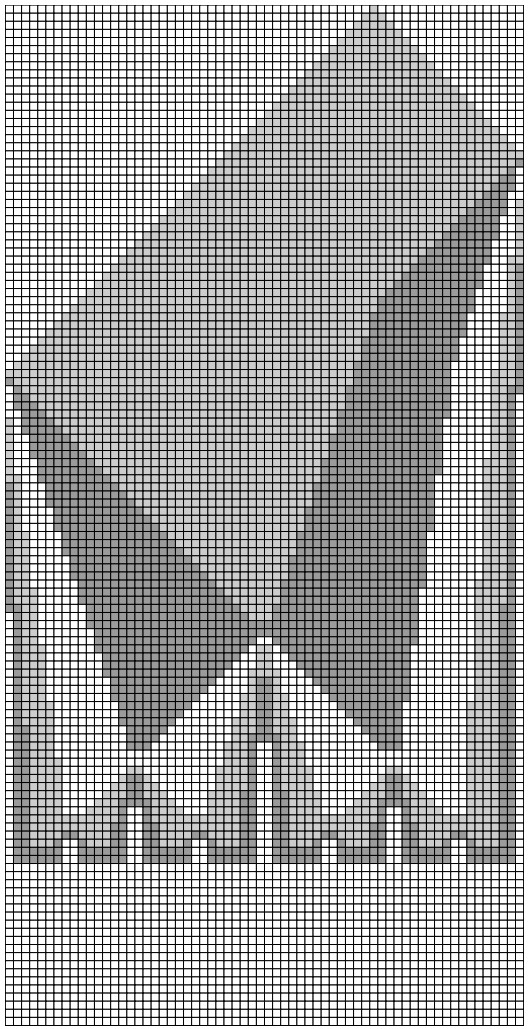
Demo 3

Space-Time Diagrams for different General's positions



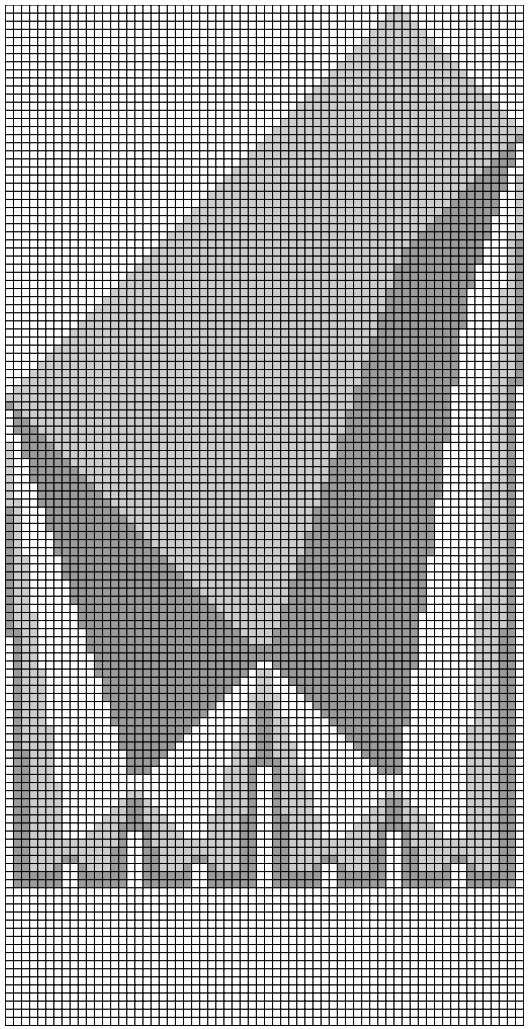
Demo 3

Space-Time Diagrams for different General's positions



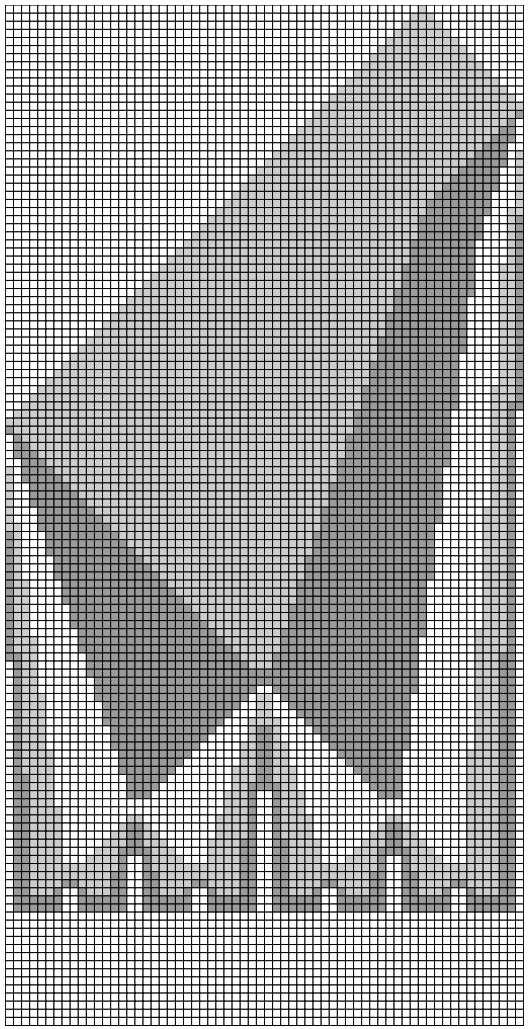
Demo 3

Space-Time Diagrams for different General's positions



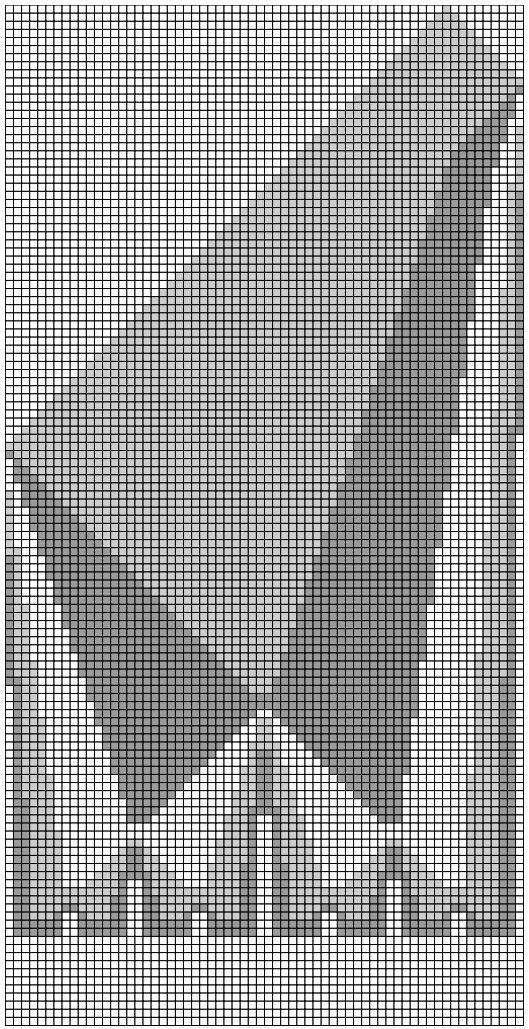
Demo 3

Space-Time Diagrams for different General's positions



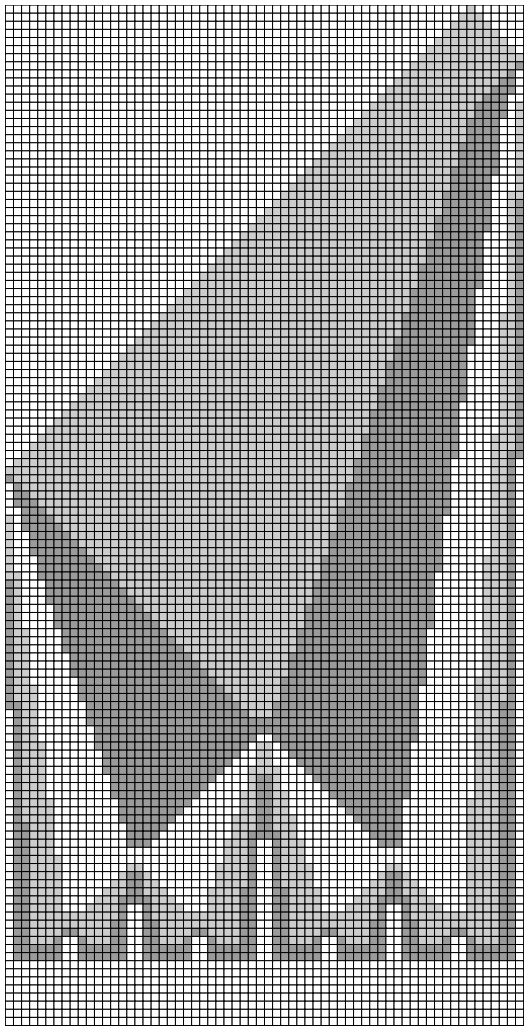
Demo 3

Space-Time Diagrams for different General's positions



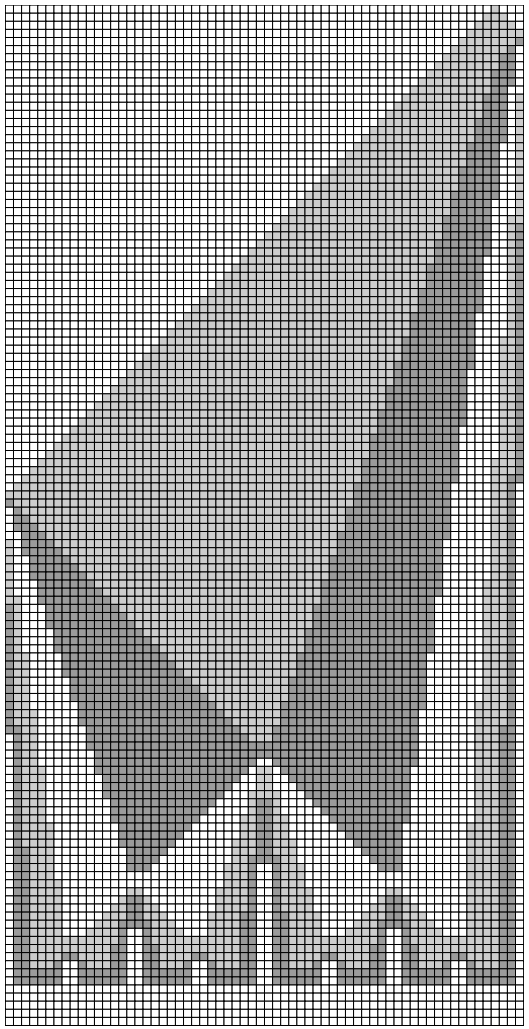
Demo 3

Space-Time Diagrams for different General's positions



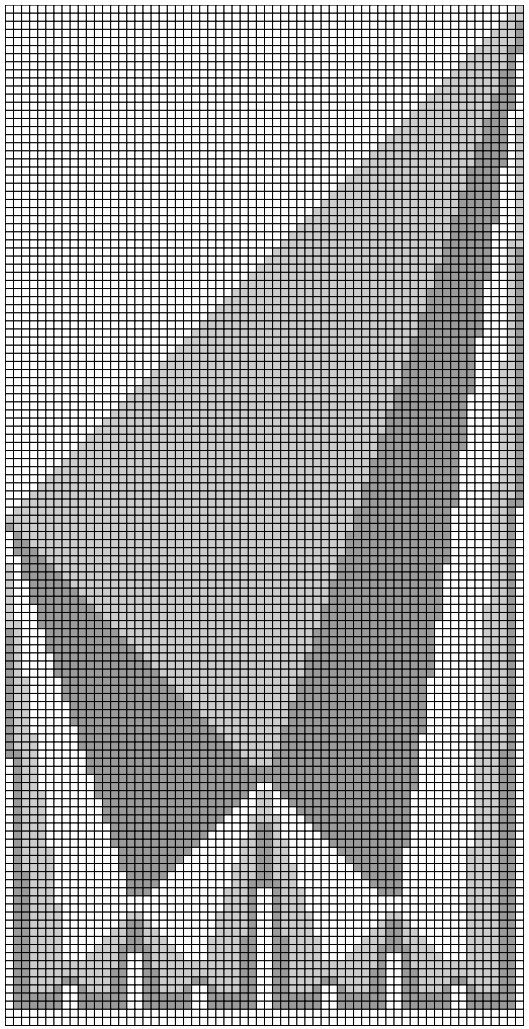
Demo 3

Space-Time Diagrams for different General's positions



Demo 3

Space-Time Diagrams for different General's positions



Demo 3

From Infinite to Finite

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	3	2	2	1	1	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	3	3	2	2	1	1	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	3	3	2	2	1	1	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	4	3	3	2	2	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	4	4	3	3	2	2	1	1	0	0	0	0	0	0
0	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	4	4	3	3	2	2	1	1	0	0	0	0	0
0	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	5	4	4	3	3	2	2	1	1	0	0	0	0
0	1	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	5	5	4	4	3	3	2	2	1	1	0	0	0
0	1	2	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	5	5	4	4	3	3	2	2	1	1	0	0
0	1	2	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	6	5	5	4	4	3	3	2	2	1	1	0
0	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	6	6	5	5	4	4	3	3	2	2	1	0
0	1	2	2	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	6	6	5	5	4	4	3	3	2	1	0
0	1	2	3	2	1	0	0	0	0	0	0	0	0	0	0	0	1	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	7	6	6	5	5	4	4	3	2	1	0
0	1	2	3	2	1	1	0	0	0	0	0	0	0	0	0	1	1	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	7	7	6	6	5	5	4	3	2	1	0
0	1	2	3	2	2	1	0	0	0	0	0	0	0	0	1	1	1	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	7	7	6	6	5	4	3	2	1	0
0	1	2	3	3	2	1	0	0	0	0	0	0	0	1	1	2	1	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	8	7	7	6	5	4	3	2	1	0
0	1	2	3	3	2	1	1	0	0	0	0	0	1	1	2	2	1	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	8	8	7	6	5	4	3	2	1	0
0	1	2	3	3	2	2	1	0	0	0	0	1	1	2	2	2	1	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	3	3	2	1	0	0	0	1	1	2	2	3	2	1	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	4	3	2	1	1	0	1	1	2	2	3	3	2	1	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	4	3	2	2	1	0	1	2	2	3	3	3	2	1	0
0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	4	3	3	2	1	0	1	2	3	3	4	3	2	1	0
0	1	2	1	0	0	0	0	1	0	1	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	4	4	3	2	1	0	1	2	3	4	4	3	2	1	0
0	1	2	1	0	0	0	1	1	0	1	1	0	0	0	1	1	1	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	4	4	3	2	1	0	1	2	3	4	4	3	2	1	0
0	1	2	1	0	0	1	1	1	0	1	1	1	0	0	1	2	1	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	4	4	3	2	1	0	1	2	3	4	4	3	2	1	0
0	1	2	1	0	0	1	2	1	0	1	2	1	0	0	1	2	1	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	4	4	3	2	1	0	1	2	3	4	4	3	2	1	0
0	1	2	1	0	0	1	2	1	0	1	2	1	0	0	1	2	1	0
0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	4	4	3	2	1	0	1	2	3	4	4	3	2	1	0
0	1	2	1	0	0	1	2	1	0	1	2	1	0	0	1	2	1	0
0	1	0	1	0	0	1	0	1	0	1	0	1	0	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Infinite to Finite

0	1	2	3	4	5	6	7	8	9	8	7	6	5	4	3	2	1	0
0	1	2	3	4	4	3	2	1	0	1	2	3	4	4	3	2	1	0
0	1	2	1	0	0	1	2	1	0	1	2	1	0	0	1	2	1	0
0	1	0	1	0	0	1	0	1	0	1	0	1	0	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

What about bidimensionnal cellular spaces ?

First naive idea works

0	1	2	3	3	2	1	0
0	1	2	3	3	2	1	0
0	1	2	3	3	2	1	0
0	1	2	3	3	2	1	0
0	1	2	3	3	2	1	0

(c) Horizontal

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0

(d) Vertical

All left neighborhood are left neighbors!

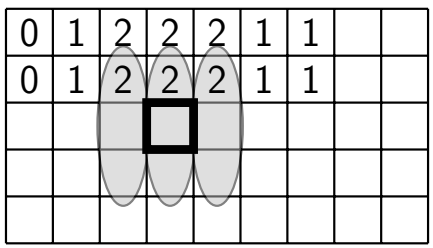
0	1	2	3	3	2	1	0
0	1	2	3	3	2	1	0
0	1	2	3	3	2	1	0
0	1	2	3	3	2	1	0
0	1	2	3	3	2	1	0

(e) Horizontal

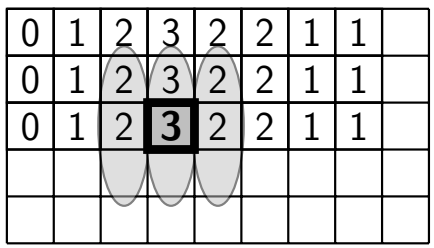
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0

(f) Vertical

All left neighborhood are left neighbors!



(g) at time t



(h) at time $t + 1$

All left neighborhood are left neighbors!

$$\mathrm{brd}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \exists i \in \mathrm{I};\ c + i \notin \mathrm{S},$$
$$\mathrm{ins}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \forall i \in \mathrm{I};\ c + i \in \mathrm{S},$$

$$\mathrm{dst}^\ell_{t+1}(c) = \begin{cases} 1 + \min_{i \in \mathrm{I}}\{\mathrm{dst}^\ell_t(c+i)\} & \text{if } \mathrm{ins}^\ell_{t+1}(c) \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{sta}^\ell_{t+1}(c) = \bigvee \begin{cases} \mathrm{brd}^\ell_{t+1}(c) \\ \exists i \in \mathrm{I}; \\ \qquad \mathrm{dst}^\ell_{t+1}(c) = 1 + \mathrm{dst}^\ell_t(c+i) \wedge\ \mathrm{sta}^\ell_t(c+i). \end{cases}$$

All left neighborhood are left neighbors!

$$\mathrm{brd}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \exists i \in \mathrm{I};\ c + i \notin \mathrm{S},$$
$$\mathrm{ins}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \forall i \in \mathrm{I};\ c + i \in \mathrm{S},$$

$$\mathrm{dst}^\ell_{t+1}(c) = \begin{cases} 1 + \min_{i \in \mathrm{I}}\{\mathrm{dst}^\ell_t(c+i)\} & \text{if } \mathrm{ins}^\ell_{t+1}(c) \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{sta}^\ell_{t+1}(c) = \bigvee \begin{cases} \mathrm{brd}^\ell_{t+1}(c) \\ \exists i \in \mathrm{I}; \\ \qquad \mathrm{dst}^\ell_{t+1}(c) = 1 + \mathrm{dst}^\ell_t(c+i) \wedge\ \mathrm{sta}^\ell_t(c+i). \end{cases}$$

All left neighborhood are left neighbors!

$$\mathrm{brd}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \exists i \in \mathrm{I};\ \forall \delta \in \nu^i;\ c+\delta \notin \mathrm{S},$$

$$\mathrm{ins}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \forall i \in \mathrm{I};\ \exists \delta \in \nu^i;\ c+\delta \in \mathrm{S},$$

$$\mathrm{dst}^\ell_{t+1}(c) = \begin{cases} 1 + \min_{i\in \mathrm{I}}\{\mathrm{dst}^\ell_t(c+i)\} & \text{if } \mathrm{ins}^\ell_{t+1}(c) \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{sta}^\ell_{t+1}(c) = \bigvee \begin{cases} \mathrm{brd}^\ell_{t+1}(c) \\ \exists i \in \mathrm{I}; \\ \qquad \mathrm{dst}^\ell_{t+1}(c) = 1 + \mathrm{dst}^\ell_t(c+i) \wedge\ \mathrm{sta}^\ell_t(c+i). \end{cases}$$

All left neighborhood are left neighbors!

$$\mathrm{brd}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \exists i \in \mathrm{I};\ \forall \delta \in \nu^i;\ c+\delta \notin \mathrm{S},$$
$$\mathrm{ins}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \forall i \in \mathrm{I};\ \exists \delta \in \nu^i;\ c+\delta \in \mathrm{S},$$

$$\mathrm{dst}^\ell_{t+1}(c) = \begin{cases} 1 + \min_{i\in \mathrm{I}}\{\ \mathrm{dst}^\ell_t(c+i)\ \} & \text{if } \mathrm{ins}^\ell_{t+1}(c) \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{sta}^\ell_{t+1}(c) = \bigvee \begin{cases} \mathrm{brd}^\ell_{t+1}(c) \\ \exists i \in \mathrm{I}; \\ \qquad \mathrm{dst}^\ell_{t+1}(c) = 1 + \mathrm{dst}^\ell_t(c+i) \wedge\ \mathrm{sta}^\ell_t(c+i). \end{cases}$$

All left neighborhood are left neighbors!

$$\mathrm{brd}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \exists i \in \mathrm{I};\ \forall \delta \in \nu^i;\ c + \delta \notin \mathrm{S},$$
$$\mathrm{ins}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \forall i \in \mathrm{I};\ \exists \delta \in \nu^i;\ c + \delta \in \mathrm{S},$$

$$\mathrm{dst}^\ell_{t+1}(c) = \begin{cases} 1 + \min_{i \in \mathrm{I}} \{ \max_{\delta \in \nu^i} \{ \mathrm{dst}^\ell_t(c + \delta) \} \} & \text{if } \mathrm{ins}^\ell_{t+1}(c) \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{sta}^\ell_{t+1}(c) = \bigvee \begin{cases} \mathrm{brd}^\ell_{t+1}(c) \\ \exists i \in \mathrm{I}; \\ \qquad \mathrm{dst}^\ell_{t+1}(c) = 1 + \mathrm{dst}^\ell_t(c + i) \wedge\ \mathrm{sta}^\ell_t(c + i). \end{cases}$$

All left neighborhood are left neighbors!

$$\mathrm{brd}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \exists i \in \mathrm{I};\ \forall \delta \in \nu^i;\ c + \delta \notin \mathrm{S},$$
$$\mathrm{ins}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \forall i \in \mathrm{I};\ \exists \delta \in \nu^i;\ c + \delta \in \mathrm{S},$$

$$\mathrm{dst}^\ell_{t+1}(c) = \begin{cases} 1 + \min_{i \in \mathrm{I}} \{ \max_{\delta \in \nu^i} \{ \mathrm{dst}^\ell_t(c + \delta) \} \} & \text{if } \mathrm{ins}^\ell_{t+1}(c) \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{sta}^\ell_{t+1}(c) = \bigvee \begin{cases} \mathrm{brd}^\ell_{t+1}(c) \\ \exists i \in \mathrm{I}; \\ \qquad \mathrm{dst}^\ell_{t+1}(c) = 1 + \mathrm{dst}^\ell_t(c + i) \wedge\ \mathrm{sta}^\ell_t(c + i). \end{cases}$$

All left neighborhood are left neighbors!

$$\mathrm{brd}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \exists i \in \mathrm{I};\ \forall \delta \in \nu^i;\ c + \delta \notin \mathrm{S},$$
$$\mathrm{ins}^0_{t+1}(c) = \mathrm{input}_{t+1}(c) \wedge \forall i \in \mathrm{I};\ \exists \delta \in \nu^i;\ c + \delta \in \mathrm{S},$$

$$\mathrm{dst}^\ell_{t+1}(c) = \begin{cases} 1 + \min_{i \in \mathrm{I}} \{ \max_{\delta \in \nu^i} \{ \mathrm{dst}^\ell_t(c + \delta) \} \} & \text{if } \mathrm{ins}^\ell_{t+1}(c) \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{sta}^\ell_{t+1}(c) = \bigvee \begin{cases} \mathrm{brd}^\ell_{t+1}(c) \\ \exists i \in \mathrm{I}; \quad \exists \delta \in \nu^i; \\ \qquad \mathrm{dst}^\ell_{t+1}(c) = 1 + \mathrm{dst}^\ell_t(c + \delta) \wedge\ \mathrm{sta}^\ell_t(c + \delta). \end{cases}$$

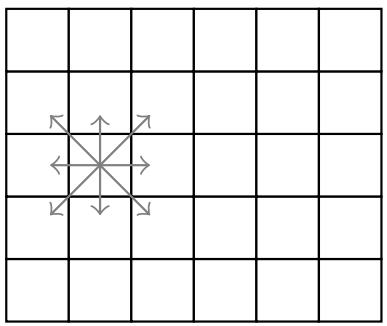
Is there any improvements?

Is there any improvements?

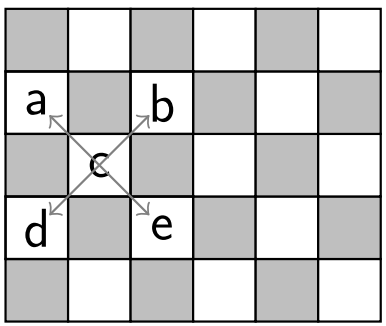
Yes, holes

Demo 4

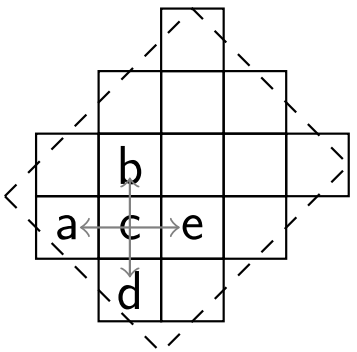
What about different neighborhoods?



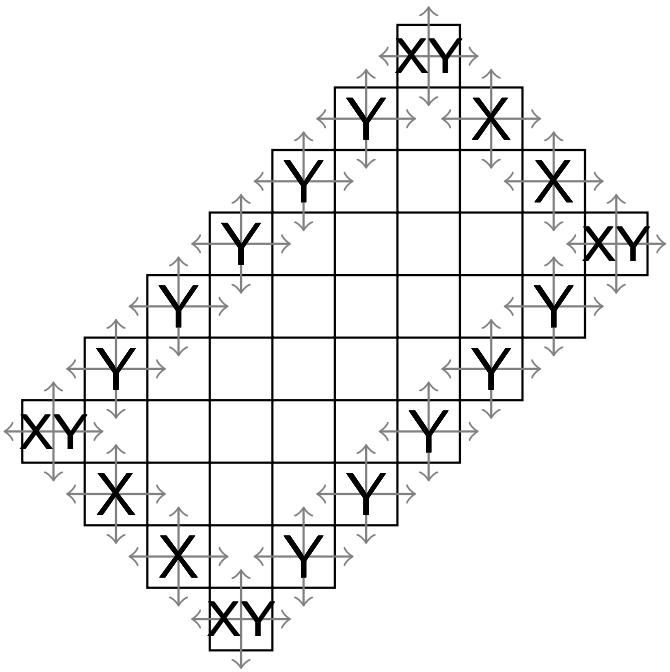
(i) Moore neighborhood

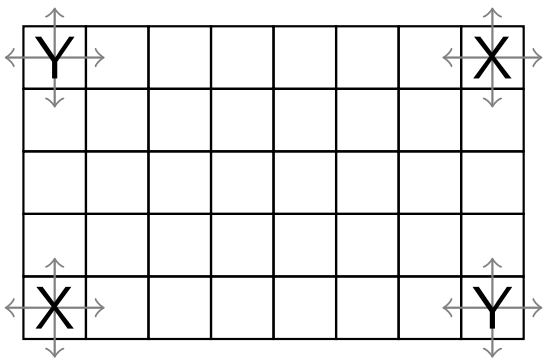


(j) Moore on a chessboard



(k) von Neumann on a grid


XY
Y
X
Y
X
Y
XY
Y
Y
Y
Y
XY
Y
X
Y
X
Y
XY


Y
X
X
Y

. . .